# All-day Program Aeromodelling 

## German Originaltitel:

Ganztagsangebot Modellflug- Ein Begleitheft für den Unterricht https://www.mfc-rossendorf.de/ > News > Begleitheft > Schrift


Accompanying booklet for lessons

## by

Hans Langenhagen

This paper was written in connection with the all-day program Aeromodelling at the high school Dresden-Bühlau. It was started in September 2009 and finished at the beginning of the holidays in July 2019.

Quote Mike Colling:

## Why Have an Education Programme?

That is a question that I have been asked more than once and the answer is for a number of reasons. Building and flying model aircraft is a most enjoyable thing to do, I would not do it if I did not enjoy it and I would like more young people to take up the sport.
Model aircraft flying is probably unique in the airsports in having no age limit. It can cost less than any other airsports and can be carried out in just about any reasonable free space both indoors and out. It can be carried out at whatever level you wish from club flying on a Sunday to competing in World Championships.

Model building and flying can be much, much more. For a youngster it can open many new doors to the large world of aviation and technology. In Britain, the BMFA in collaboration with British Aerospace uses a model building and flying contest in schools as a way to promote interest in Engineering and Technology through flight. The BMFA also promotes model design building and flying with its University Challenge.
A large number of Universities are realising the value of using a model design and building project as a good low cost method to train students in how to problem solve and manage a project.

Colleges of Further education are using models to tempt some young students, about to leave school, to look at a career in aircraft management as an option. Model building and flying can stimulate a youngster's mind and help to keep their desire to enter their chosen airsport when finances allow and the correct age is attained.

| Mike Colling was editor of BMFA News magazine (U.K.) for many years. |
| :--- |
| But he was also active internationally, heading the FAI-CIAM |
| SubCommittee Education/Technical Experts for a number of years. |
| The text is an Extract from CIAM-flyer Issue \#10/March 2000 |
| https://www.fai.org/sites/default/files/documents/cf2000.pdf |
| M. Colling kindly authorises the author to include this in his brochure. |

## Greeting

"Only flying is nicer" - who doesn't know this saying and who hasn't tried to fly with small model airplanes as a child. Everyone very quickly made the experience that in addition to skill in flying, manual skills and technical understanding are also required.

With this brochure, Hans Langenhagen has created a primer on model aircraft construction and model flight. Almost all areas of this very exciting hobby are illuminated, the work steps involved in building are described in detail and the theoretical background is sometimes very intensively examined.
The fact that a phase of the author's life was also put down on paper shows the enthusiasm with which he taught model flying to children, young people and adults from scratch for more than 50 years in the Rossendorf model flying club. The numerous pictures make you want to build model airplanes. At the same time, they are illustrious contemporary witnesses over the decades.

The all-day offer for schools finds a wide space in the presentation. This represents a special form of school education in which school physics should become clearer and more "tangible" for the children based on experience. What is better for this than a model airplane. It is to the credit of Hans Langenhagen that he dedicated himself to this task, giving the children fun experimenting and, quite incidentally, also winning new club members.
The harmony of theory and practice is constantly postulated and finds constant confirmation in the many examples presented. While in the beginnings of this hobby there are seemingly trivial considerations that have to be implemented on the model, the theoretical background increases with larger models to quite imposing theoretical frameworks. So there is something for everyone: for the beginner who wants to get a flying experience quickly and also for the advanced modeler who wants to recognize and try out the subtleties (and also nastiness) of aerodynamics and flight mechanics.

Unforgotten and not unmentioned at this point is Rudi Zimmermann, the good soul of the association and permanent contact point in the workshop. He, too, would have his work confirmed with this brochure. Unfortunately, he could not live to see its completion.
Let us consider this documentation as a tribute to the life's work of a recreational modeler. May it delight many readers with suggestions and technical solutions, so that quite a few technology enthusiasts still see model flying as a meaningful leisure activity.

Prof. Dr. Norbert Brückner

Foreword
Schools in Germany do not only offer the regular curriculum. Rather, it is common that in addition to this, students are offered education in areas that might also interest them. Examples include the school choir or playing musical instruments individually or instruction in a technical field. And here we aero modeller have the chance to recruit new blood for our club. Using the example of a cooperation between the Modellfluglub Rossendorf e.V. and a nearby high school in Dresden, this booklet shows that such a cooperation can be also a bridge to the school syllabus as well to small projects in the context of physics and information technology (IT).

The main topic in the first part of the paper is the practical approach to model airplane building; from the simple to the more difficult. Already with the simple models basic characteristics are described, which allow statements about the flight quality later on. Notes on RC-Training will also be included.

Theory is applied within the second part: with the help of the basic properties studied in simple models, further important parameters can be determined and confirmed by simple measurements. Additionally guidelines fort the design of a model are given, including calculation examples on practically applied models. Commercially available motor gliders of different performance classes are recalculated exemplarily regarding both their flight characteristics as well as their motor-propeller configuration.

Thus, this report could offer something new to the student and likewise be useful to the instructor for reference. For both use cases, the manuscript would have been worthwhile.

## Table of Contents

|  |  | page |
| :---: | :---: | :---: |
| 1.0 Offer opening | Explanation of the syllabus | 01\| |
| Aeromodelling and aviation | Lecture on the history and physics of aviation |  |
| Rossendorf light aircraft model | construction and first flight tests | 02 \| |
| 2.0 Balsa glieder JIMMY | Start of construction | 04 \| |
|  | Calculation of wing suface and wing loading |  |
|  | Center of gravity adjustment, flight-test | 05 |
|  | flight parameters, air forces on the flight model | 06 |
|  | Lift and drag force | 07 |
|  | Why an airplane flies | 08 \| |
| 3.0 Electric Powered Sailplane CLIMAXX | Kit Opening, Explanation of Construction | 10 \| |
|  | Why a wing must also be solidly built | 12 I |
|  | Which glue for which joint? | 14 \| |
|  | Tail plane construction | 15 \| |
| 4.0 Remote control exercises | We control club models | 16 \| |
|  | Longitudinal control, flare-out path on landing | 17 \| |
|  | Turning control | 18 \| |
|  | Rudder adjustments on the tail unit | 19 \| |
|  | covering with iron-on foil | 21\| |
| 5.0 What we can calculate with air forces |  | 24 \| |
| 5.1 Calculation with coefficients | Examples for the maximum lift coefficient | 26 \| |
| 5.2 The stationary flight conditions | glide flight /sink polar; horizontal flight; climb flight / climb polar | 28 \| |
| 5.3 Propulsion |  |  |
| 5.3-1 | Thrust, power, efficiency | 38 \| |
| 5.3-2 | Drive components / battery | 41 \| |
| 5.3-3 | / motor and propeller | 46 \| |
| 5.3-4 | / speed controller | 54 \| |
| 5.3-5 | / selection criteria | $56 \mid$ |
| 6.0 Strength |  |  |
| 6.1 | Material parameter | 58 \| |
| 6.2 | Load cases | 59 \| |
| 6.3 | Forces in the wing | 63 \| |
|  | Development steps indoor training model | 68 \| |
| 7.0 Design of a flight model |  |  |
| 7.1 | Purpose | 69 \| |
| 7.2 | Wing geometry | 72 \| |
| 7.3 | Setting angle difference | 74 \| |
| 7.4 | Neutral point and C.G. position, longitudinal stability | 78 \| |
| 7.5 | Drive selection | 78 \| |
| 8.0 Selected topics |  |  |
| 8.1 | Measurements on pusher low wing | 831 |
| 8.2 | Motorglider Luxx on launch trolley | 85 |
| 8.3 | Take-off with tricycle landing gear | 88 |
| 8.4 | Speed polar as Java program | $90 \mid$ |
| 8.5 | Tolerance analysis | 91 \| |
| 8.6 | Servo load | 92 \| |
| 8.7 | Solar drive - how many cells are needed? | 97 \| |
| 8.8 | Take-off process of a powered model | 991 |
| 8.9 | F3K hand-launch, is lightweight construction worthwhile? | 104\| |
| 8.10 | Summary, Acknowledgements | 106 |
| 9.1 Appendix | Calculation of the moment replacement wing | 110 |
|  | Literature | 113 \| |

## Examples of measurements and calculations

| expl. | page | Topic |
| :---: | :---: | :---: |
| Nr. |  |  |
|  |  | From the teaching material |
|  | 9 | Estimation of lift and drag force of balsa glider Jimmy;; typical glide ratios for balsa glider, F3B model, glider and commercial aircraft. |
| 5.1 | 26 | Computational estimation of lift coefficients of a) an indoor training model, b) an F3J model, c) the glider JANTAR, d) the Airbus A320 |
|  | 32 | Difference: minimum rate of descent and best glide ratio; descent polars. |
| 5.2 | 39 | Uniform motion and uniformly accelerated motion; estimation: (a) the propulsive force (the thrust) $\mathrm{F}_{\text {th }}$ to be delivered by the propeller and the input (=electric) power $\mathrm{Pel}_{\text {el }}$ to reach the take-off speed at a given distance; <br> b) the required input power for the desired take-off speed of an F5J model, c) the required thrust of the A320 during take-off and cruise. |
|  | 42 | Battery: discharge time vs. discharge current |
|  | 53 | Difference: maximum rate of climb and maximum angle of climb; climb polar curve |
| 5.3 | 55 | powered sailplane: minimum input power for a DAeC youth competition task |
|  | 60 | Load cases rope-high take-off and looping |
| 6.1 | 64 | Dimensioning of spar belts |
| 6.2 | 66 | Dimensioning of carbon connectors between wing sections |
|  | 67 | Carbon spars for stiffening simple polystyrene airfoils of an indoor training model, profiled with curved plate Gö417a |
|  | 72 | Angle of attack and center of gravity |
|  | 78- | Dimensioning/Recalculation Engine power and propeller values of industrial models: |
| 8.4 | 92 | Servo load. Measurement of flap forces on glider model, calculation of rudder forces on aerobatic model Mx2. |
|  | 97 | Solar drive: how many cells are necessary? Example of drive configuration |
|  | 104 | F3K Spin take-off, is lightweight construction worthwhile? |
|  | 110 | Calculation of momentum replacement wing |
|  |  | Student work |
|  | 83 85 | Uniform motion, indoor flight. Measurement: slow and fast flight, find associated working points on model polar curve. |
| 8.2 | 85 | uniformly accelerated motion, take-off procedure, powered glider on take-off trolley; Measurement and estimation: take-off speed, lift coefficient, angle of attack at the moment of Itake-off |
|  | 88 | take-off with tricycle landing gear, estimation: what wing angle of attack is necessary so that at desired speed the model rotates around the main undercarriage with "elevator-pull"? |
| 8.4 | 90 | Speed polar as Java program |

## List of symbols

| Symbol | Unit | Meaning | page | fig | Eq |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A |  | Aspect ratio | 30 |  | 5.7 |
| a | $\mathrm{m} / \mathrm{s}^{2}$ | acceleration | 84 | 8.2 |  |
| ac |  | Aerodynamic Center |  |  |  |
| $\alpha$ <br> $\alpha_{w}$ $\alpha_{\text {hs }}$ $\alpha_{A}$ |  | Angle of attack of the of wing of the horizontal stabiliser of the wing, taking into account the wing aspect ratio A | $\begin{aligned} & 25 \\ & 73 \\ & 71 \end{aligned}$ | $\begin{aligned} & 5.2 \\ & 7.7 \\ & 7.4 \end{aligned}$ | 7.3 |
| $\mathrm{B}=\mathrm{m} / \mathrm{S}_{\mathrm{w}}$ | $\mathrm{gm} / \mathrm{dm}^{2}$ | wing load, Tab. 2.1 | 4 |  |  |
| Beu |  | corrective factor for calculation auf xN | $\begin{aligned} & \hline 75 \\ & 76 \\ & \hline \end{aligned}$ | 7.12 | 7.5b |
| b | m | wing span | 5 | 2.2 |  |
| C | As | rechargeable battery capacity |  |  |  |
| c | mm | airfoil chord; chord length, | 25 | 5.2 |  |
| cg |  | center-of-gravity |  |  |  |
| $\begin{array}{ll} \mathrm{c}_{\mathrm{d}} & \\ & \mathrm{c}_{\mathrm{dp}} \\ & \mathrm{c}_{\mathrm{do}} \\ & \mathrm{c}_{\mathrm{di}} \\ & \mathrm{c}_{\mathrm{dh}} \\ & \mathrm{c}_{\mathrm{dtot}} \\ \hline \end{array}$ |  | Drag coefficient <br> - airfoil drag <br> - airfoil drag minimum <br> - induced drag <br> - harmful drag <br> - of the model | $\begin{aligned} & 25 \\ & 30 \\ & 30 \\ & 30 \\ & 32 \\ & 32 \\ & \hline \end{aligned}$ | 5.2 | $\begin{aligned} & 5.1 / 2 \\ & \\ & 5.8 \\ & 5.6 \mathrm{c} \\ & 5.6 \\ & \hline \end{aligned}$ |
| cı |  | Lift coefficient | 25 | 5.2 | 5.1/2 |
| $\mathrm{c}_{\mathrm{m}}$ |  | Moment coefficient | 75 | 7.10 |  |
| $\chi_{0}$ |  | Downwash angle Chi | 73 |  | 7.4a |
| D |  | diameter propeller | $\begin{array}{\|l\|} \hline 78 \\ 80 \\ \hline \end{array}$ |  | $\begin{aligned} & \hline 7.7 \\ & 7.10 \\ & \hline \end{aligned}$ |
| E |  | lift -to- drag -ratio $\mathrm{E}=1 / \varepsilon$ | 6 |  | 5.5a |
| $\varepsilon$ |  | drag-to-lift-ratio; $\varepsilon=1 / \mathrm{E}$ |  |  |  |
| EWD | deg | angle of setting | 72 | 7.7 |  |
| F |  | Force | 28 | 5.4 |  |
| $\begin{aligned} & \mathrm{F}_{\mathrm{L}} \\ & \mathrm{~F}_{\mathrm{D}} \\ & \mathrm{~F}_{\mathrm{W}} \\ & \mathrm{~F}_{\mathrm{T}} \\ & \mathrm{~F}_{\mathrm{C}} \\ & \mathrm{~F}_{\mathrm{RF}} \end{aligned}$ |  | lift Force drag force weight force thrust centrifugal rolling friction | $\begin{array}{\|l} 28 \\ 34 \\ 59 \\ 101 \end{array}$ | 5.4 <br> 5.10 <br> 6.1 <br> 8.23 | $\begin{aligned} & 5.3 \\ & 5.28 \\ & 8.19 \\ & \hline \end{aligned}$ |
| $\phi$ |  | wing dihedral angle | 66 | 6.16b | 7.5d |

$\left.\begin{array}{|l|l|l|l|l|l|}\hline \text { Symbol } & \text { Unit } & \text { Meaning } & \text { page } & \text { fig } & \text { Eq } \\ \hline \mathrm{g} & & & & & \\ \hline \gamma & & \text { acceleration of gravity } & & & \\ \hline \gamma_{\mathrm{c}} & & \text { angle of fligth path } \\ \text { climb angle p. 36, 37 }\end{array}\right)$

| Symbol | Unit | Meaning | page | fig | Eq |
| :---: | :---: | :---: | :---: | :---: | :---: |
| P | W | Power |  |  |  |
| Pel | W | electric power supplied to the motor |  |  |  |
| $\mathrm{P}_{\text {dis }}$ | W | dissipation - Verlustleistung - |  |  |  |
| $\mathrm{P}_{\text {mech }}$ | W | mechanical power loss f.i. by friction |  |  |  |
| $\mathrm{P}_{\text {sh }}$ | W | mech. power supplied by the motor shaft |  |  |  |
| $\mathrm{P}_{\mathrm{pr}}$ | W | power absorbed by propeller | 50 | 5.33 | 5.25 |
| $\mathrm{P}_{\text {out }}$ | W | mech. power, supplied by the propeller |  |  |  |
| $\mathrm{P}_{\text {sc_ }}$ req | W | requested power, solar drive | 98 | 8.21 |  |
| $\mathrm{P}_{\text {sc_del }}$ | W | delivered power, solar cells | 98 | 8.21 |  |
| Q |  | angular velocity (pitch rate) about airplane's y-axis |  |  |  |
| R |  | internal resistance of an electric motor | $\begin{aligned} & \hline 47 \\ & 81 \end{aligned}$ | 5.27 | 7.12 |
| $\mathrm{R}_{\mathrm{i}}$ |  | internal resistance of a recharg. battery | 42 | 5.20 |  |
| Re |  | Reynolds number |  |  |  |
| $\begin{aligned} & r_{h} \\ & r_{v} \end{aligned}$ |  | horizontal vertical tail moment arm | $\begin{aligned} & 75 \\ & 77 \\ & \hline \end{aligned}$ | 7.10 | $\begin{aligned} & \hline 7.5 \mathrm{c} \\ & 7.5 \mathrm{~d} \\ & \hline \end{aligned}$ |
| $\rho$ | $\mathrm{kg} / \mathrm{m}^{3}$ | air density $1.23 \mathrm{~kg} / \mathrm{m}^{3}$ | 24 |  | 5.1/2 |
| R |  | radius of cylinder |  |  |  |
| R |  | propeller radius | 48/9 |  | 5.29 |
| RPV |  | remote piloted vehicle |  |  |  |
| R/C |  | remote control |  |  |  |
|  |  |  |  |  |  |
| $\mathrm{S}_{\mathrm{w}}$ | $\mathrm{dm}^{2}$ | Wing planform area | 24 |  | 5.1/2 |
| SM |  | Stability Margin | 76 |  | 7.6a |
| $\mathrm{S}_{\text {lo }}$ | m | space at the moment of lift-off | 87 |  | 8.2 |
| ${ }^{\sigma b}{ }_{\text {Ob_perm }}$ | $\begin{aligned} & \mathrm{N} / \mathrm{mm}^{2} \\ & \mathrm{~N} / \mathrm{mm}^{2} \end{aligned}$ | bending stress permitted bending | $\begin{aligned} & 63 \\ & 64 \end{aligned}$ |  |  |
| $\sigma$ ts | $\mathrm{N} / \mathrm{mm}^{2}$ | tensile strength | 58 |  |  |
| $\sigma_{\text {ps }}$ | $\mathrm{N} / \mathrm{mm}^{2}$ | compressive strength | 58 |  |  |
| $\sigma_{\text {b }}$ | km | breaking length | 58 |  |  |
|  |  |  |  |  |  |
| tD | sec; hr | discharge time of a rechargeable battery | 41 | 5.18 | 5.18 |
| to | sec | time at the moment of lift-off | 87 |  | 8.2 |
| T | N | Thrust |  |  |  |
| TO |  | relatively Tolerance, deviation of rate value | $\begin{aligned} & \hline 91 \\ & 92 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 8.7 \\ & 8.8 \\ & \hline \end{aligned}$ |
|  |  |  |  |  |  |
| v | m/s | velocity | 28 | 5.4 | 5.3 |
| Vlo | $\mathrm{m} / \mathrm{s}$ | at the moment of lift-off | 87 |  |  |
| V cruise | $\mathrm{m} / \mathrm{s}$ | cruise velocity | 83 |  |  |
| $\mathrm{V}_{\infty}$ | $\mathrm{m} / \mathrm{s}$ | free-stream velocity | 8 | 2.6/7 |  |
| $\mathrm{V}_{\text {sr }}$ | $\mathrm{m} / \mathrm{s}$ | sink rate, rate of descent/sink | 28 | 5.4 | 5.4 |
| Vcr | $\mathrm{m} / \mathrm{s}$ | climb rate | 36 | 5.13 | 5.14 |
| $\mathrm{V}_{\mathrm{pr}}$ | $\mathrm{m} / \mathrm{s}$ | air column speed behind the propeller | 49 |  | 5.24 |


| Symbol | Unit | Meaning | page | fig | Eq |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| $\mathrm{V}_{\mathrm{H}}$ |  | horizontal tail stabilizer volume | 75 |  | 7.5 c |
| $\mathrm{V}_{\mathrm{V}}$ |  | vertical tail stabilizer volume | 77 |  | 7.5 d |
|  |  |  |  |  |  |
| W | N | model weight, W = mass*g |  |  |  |
| $\mathrm{W}_{\text {spar }}$ | $\mathrm{mm}^{3}$ | resistance of spar | 64 | 6.10 | 6.9 b |
|  |  | Chi (from „c") |  |  |  |
| $\chi$ |  | position of Neutral Point model | 73 | 7.7 | 7.4 b |
| $\mathrm{x}_{\mathrm{N}}$ | zeta (from „x") |  | 75 | 7.5 c |  |
| $\xi$ |  | spanwise distance along a wing span from <br> the center |  |  | 7.4 b |
| y |  |  |  |  |  |

The start of the offer in the Rossendorf model making workshop, 1 September 2009
Mr. Langenhagen explains the aim of the GT offer: starting with building simple models, we will approach building different remote control models for training purposes. Often we will build from the kit, i.e. many parts are already prefabricated.
Parallel to the construction work, we will also learn about the physics of flight and how to control a model aircraft remotely: on the simulator in the workshop or with the club's own models (using teacher-student remote control technology) in the gym or on the flying field at Weißig.


The founders of the All-day Program Model Flight: R. Zimmermann (top right, †18.05.21) and H. Langenhagen. They were supported by K. Wenzel in 2011/13, F. Schade in 2014/16 and A. Scheffler in 2016/19. "Fire missions" were also performed by W. Dienel, KH. Helling, A. Lorenz and M. Müller
1.0 We start building the "Rossendorf light glider", a polystyrene glider.


First we build a polystyrene glider. We can then also test it in the corridor of the clubhouse. We are Kelvin from class 5.2, Stefan from class 5.3 and Florian from class 7.1. Zacharias from class 5.3 will join us later.


Then we build the balsa glider CANDY, here the individual parts are pre-cut in a Balsa-board

### 2.0 We build the Balsa-Gliders Candy und Jimmy



Remove the parts from the pre-cut balsa board. They are joined, aligned and glued together as

specified in the building plan.. On the finished model we set the required position of the centre of

gravity with the help of the rocker. The "fine-tuning" is then done during the first gliding flights.. The Candies are ready and tested


Fig 2.1 Packaging of Jimmy.

- Construction kit from Graupner, the balsa parts are pre-cut and easy to break out.
- Mr. Zimmerman explains the individual assembly steps to each of us and we begin with the fuselage gluing.

The wing transitions: the centre section and the bent end sections are sanded.

- We calculate the centre of the wing in order to find the place for the support board where the wing can be fixed symmetrically to the fuselage support board with rubber rings.
- With steel balls in the fuselage-nose we move the C.G. on the position required by the construction plan.
- We check with the C.G. "balance" whether the C.G. is now at the right position.
- We have to estimate the area of the wing. To do this, we look at it as a rectangle, measure its span (from one wing tip the other) and Root Chord Length and calculate the rectangle; we get a total surface area of $4,85 \mathrm{dm}^{2}$ :


Parameter of Balsa-Glider: wing span $b$, wing area $S$ and specific wing mass $B=m / S$

Here, Mr. Langenhagen has drawn it neatly and also calculated it more precisely:


Fig 2.2 We get the wing area by adding the contents of the rectangle and two trapezoids.
Attention: for wings with Dihedral-Angle or "kink ears" the projection on the plane counts.
Wing area JIMMY
wing dimensions $\quad \mathrm{a}:=4.05 \cdot \mathrm{dm} \quad \mathrm{e}:=0.95 \cdot \mathrm{dm} \quad \mathrm{c}:=0.85 \cdot \mathrm{dm} \quad \mathrm{d}:=0.7 \cdot \mathrm{dm}$

| wing span | $\mathrm{b}:=\mathrm{a}+2 \cdot \mathrm{e}$ | $\mathrm{b}=5.95 \mathrm{dm}$ |
| :--- | :--- | :--- |
| wing section rectangular | $\mathrm{Sr}:=\mathrm{a} \cdot \mathrm{c}$ | $\mathrm{Sr}=3.442 \mathrm{dm}^{2}$ |
| wing section trapezoid | $\mathrm{Str}:=\frac{\mathrm{c}+\mathrm{d}}{2} \cdot \mathrm{e}$ | $\mathrm{Str}=0.736 \mathrm{dm}^{2}$ |
| wing area total | $\mathrm{S}:=\mathrm{Sr}+2 \cdot \mathrm{Str}$ | $\mathrm{S}=4.915 \mathrm{dm}^{2}$ |

$\rightarrow$ So we've got the surface area quite right with our estimation calculation!


Florian completes the decor, while Stefan already establishes the position of the centre of gravity cg

## We summarise the characteristics of the JIMMY:

|  |  | Antony_Jimmy | Karl_Jimmy |
| :--- | :--- | :--- | :--- |
| wing area | S | $4,92 \mathrm{dm}^{2}$ | $4,92 \mathrm{dm}^{2}$ |
| aspect ratio | A | $5,95 \mathrm{dm} / 0,85 \mathrm{dm}=7$ | 7 |
| modell mass | m | 58 g | 80 g |
| wing load | B | $59 \mathrm{~g} / 4,9 \mathrm{dm}^{2}=11,83 \mathrm{~g} / \mathrm{dm}^{2}$ | $80 \mathrm{~g} / 4,9 \mathrm{dm}^{2}=16,33 \mathrm{~g} / \mathrm{dm}^{2}$ |

Table 2.1 Mass and specific mass per unit area (wing load) of two models related to the wing area. By the way: The lower the wing loading B, the slower a model can fly; more on this in Chapter 5.2

Afterwards we go to the settlement sports field and do hand launches with the aim of determining the inherent speed and glide ratio of the models.


Fig 2.3: Hand launch to determine flight time and distance. The model only comes onto a glide path (case c), if it is released at the right angle and with precisely dosed force ... this you have to practise a bit.

Everyone is allowed to throw 5 times. We measure the flight times with the stopwatch and estimate the flight distance $\Delta$ s by counting steps. Finally, we calculate the average value from the best values of each individual:

|  | Arne | Justus | Max | Mr. Zimmerm. | average |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ejection height/m *) | 0,7+1,60 = 2,30 | 0,7+1,90 = 2,60 | 0,7+1,65 = 2,35 | 0,7+1,90 $=2,60$ |  |
| best Flight distances / m | 13 | 16 | 14 | 15 |  |
| best Flight times/ s | 4 | 5 | 4 | 4 |  |
|  |  |  |  |  |  |
| $\rightarrow$ aerodyn. quality E **) | 13/2,3 =5,7 | 16/2,60 = 6,15 | 14/2,35 = 5,95 | 15/2,60 = 5,8 | 5,9 |
| $\rightarrow$ path speedv in $\mathrm{m} / \mathrm{s}$ <br> $\mathrm{km} / \mathrm{h}$ | $\begin{aligned} 13,2 / 4 & =3,3 \\ & =11,9 \end{aligned}$ | $\begin{aligned} 16,21 / 5 & =3,24 \\ & =11,7 \end{aligned}$ | $\begin{aligned} 14,2 / 4 & =3,55 \\ & =12.8 \end{aligned}$ | $\begin{aligned} 15,22 / 4 & =3,81 \\ & =13,7 \end{aligned}$ | $\begin{gathered} 3,5 \\ 12,5 \\ \hline \end{gathered}$ |
| $\rightarrow$ rate of sink $\mathrm{V}_{\text {sr }}$ in $\mathrm{m} / \mathrm{s}$ | $2,3 / 4=0,58$ | 2,6/5 = 0,52 | 2,35/4 $=0,59$ | 2,6/4 $=0,65$ | 0,59 |

Table 2.2 Flight parameters of the Jimmy determined from the rough measurements - these are approximate values
*) We started the models from a small hill, which is about 70 cm high. To this we have to add our personal starting height, which also depends on how high we hold our arm.
**) the term glide ratio $\varepsilon$ is often used, which describes the ratio in relation to $1-$ so in the first column this would $\operatorname{read} \varepsilon=1: 5,7$. More convenient for calculations, however, is the reciprocal value $\mathrm{E}=1 / \varepsilon$

## But how do the trajectories in figure 2.3 deviate from the glide case " c "?

To answer this question, let us first recall the parachute, which we discussed in the lecture "History and Physics of Aviation" [La16] in connection with Leonardo da Vinci's inventions:

The so-called stationary speed, which in the example shown in Figure 2.4 is about $4 \mathrm{~m} / \mathrm{s}$, results from the fact that the forces acting on the falling body have "levelled off". The same pointer lengths do not mean that the jumper stops in the air, but that a descent takes place in which the speed no longer changes. We are talking about uniform, unaccelerated movement.



Fig 2.4 In a parachute jump, after a transition phase, the weight force $F_{W}=m \cdot g$ and the drag force $F_{D}$ are in "equilibrium". i.e. equal in magnitude but opposite in direction. The rate of descent, referred to as sink rate in the following, reaches a stationary value whose magnitude depends on mass m and the air drag $\mathrm{F}_{\mathrm{D}}$; the latter is directly proportional to the parachute area S .

Now let's look at the glide of a glider model. Once we have brought it onto the glide path at the "appropriate" speed, a stationary speed will also occur here, i.e. the forces are in equilibrium, Figure 2.5 . But which forces are these?


Fig 2.5 Forces during gliding flight of a glider model. More detailed information in section 5.2

We are already familiar with the weight force as the product of mass and acceleration due to gravity from parachute jumps, $F_{W}=m \cdot g$. Now it is counteracted by a force which is composed of the lift force $\mathrm{F}_{\mathrm{L}}$ generated by the aerofoil and the drag force $F_{D}$; it is therefore called the resultant force $F_{R}$. The (air) resistance force $F_{D}$, in turn, is in equilibrium with the propulsion force $F_{T}$, which arises because the model glides downwards like a ball on the inclined plane due to gravity.

From the lecture "History and Physics of Aviation" [HisAvi] we know that the lift force on an aircraft arises as a result of its movement through the medium of air. At a sufficiently high speed, a bound vortex forms around the wing, the so-called circulation. This leads to the velocity of the air flowing around the upper surface of the wing becoming greater than that of the aircraft itself. According to Bernoulli, negative pressure develops at high flow velocities and positive pressure at low flow velocities in relation to the more distant medium. The effect of these pressure differences is the lift force $F_{L}$.


Fig 2.6 On the upper side of the wing, the incoming flow velocity is added to the velocity of the circulation flow. On the underside it is subtracted.

The different flow velocities on the upper and lower wing surface cause a spatial pressure distribution along the airfoil chord $1 \mu$ as well as along the span, Figure 2.7 . As an approximation, $2 / 3$ of the total lift force of the wing is caused by negative pressure on the upper surface and $1 / 3$ by positive pressure on the lower wing surface.

However, this pressure difference wants to equalise itself, which it finds the opportunity to do at the wingtips. This pressure equalisation is accompanied by a vortex formation behind the wingtips and generates a considerable proportion of the aircraft drag in flight regimes with large angles of attack (thermal gliding, climb and landing approaches of commercial aircraft). However, it is easy to imagine that the greater the span $b$ in relation to the airfoil length $1 \mu$, the smaller the effect of this pressure equalisation on flight performance. We will come back to this in chapter 5 .
The quantities buoyancy and drag force shown as arrows in figures 2.5 and 2.6 are therefore only an idealisation of the true conditions, but this has the advantage that it can be used to calculate other quantities, such as speed, very easily.


Fig 2.7 As a result of the circulation, there is a spatial pressure distribution around the entire wing. At its ends, there is a flow from areas of higher to lower pressure.

We return to our hand launch experiments according to Figure 2.3. As we will see later, the higher the airspeed, the greater the magnitudes of the air forces $F_{L}$ and $F_{D}$. If we give the model to high an velocity $v$ by applying to much force, the lift force $F_{L}$ - and thus also the resulting force $F_{R}$ - increases strongly; figuratively speaking, the length of the force pointer $F_{R}$ in Fig. 2.5 becomes greater than that of the weight force $F_{W}$, since the latter remains constant. The model climbs steeply upwards. During this upward flight, however, the initial energy is consumed, the speed decreases again until it is so small that the circulation collapses and the wing no longer provides lift. The model tilts, picks up speed again and the game begins anew, curve a) in Figure 2.3. If we give the model to low a velocity v by applying to little force, the airfoil does not generate enough lift in the first place, the pointer length of the force $F_{R}$ remains smaller than that of the weight force $F_{W}$, the model does not come onto a glide path, curve b) in Fig. 2.3.
In the evaluation of the glider flights, Table 2.2, we encountered the parameter glide ratio $\varepsilon$ or its reciprocal value aerodynamic quality $E$. It describes the ratio of distance travelled to height loss, $E=\Delta s / \Delta h$. Figure 2.5 shows a correlation between this parameter and the flight path angle $\gamma$ : the smaller this angle, the better the glide ratio, the greater E . We also see this gliding angle - and thus also E - in the ratio of lift force to drag force: $E=F_{L} / F_{D}$.

Because the resulting force $F R$ for small angles $\gamma \leq 10^{0}$ is approximately as large as the lift force $F_{L}$, i.e. $F_{R} \approx F_{L}$, $\mathrm{F}_{\mathrm{L}} \approx \mathrm{F}_{\mathrm{G}}$ also applies. The lift force is therefore almost as great as the weight force:

$$
F_{W} \approx 65 \mathrm{~g} \cdot 9,81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}=0,64 \mathrm{~N}
$$

And because is also $E=F_{W} / F_{D}$, we can now determine the drag force of this small flying model to be

$$
F_{D} \approx \frac{F_{W}}{E}=\frac{65 \mathrm{~g}}{6} \cdot 9,81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}=0,11 \mathrm{~N}
$$

The estimation of these parameters should give us a feeling for the air forces acting on the JIMMY. Table 2.3 shows that the parameters glide ratio, velocity and sink rate are not only parameters of model aircraft. Here are the values for a glider and for a comercial aircraft ${ }^{1}$ in comparison to a balsa glider and to a model of a competition class.
We see that the aerodynamic quality of the glider is far superior. This is understandable compared to the commercial aircraft, because gliding is the essential mission for the glider. Economic considerations play no role in its design; it transports a maximum of two people at only moderate speed.

|  | balsa glider <br> Jimmy | RC soaring <br> RS3, class F3B | glider <br> JANTAR 2B | commercial aircraft <br> A 320 |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{E}=1 / \varepsilon \quad$ in $\mathrm{m} / \mathrm{s}$ | 6 | 20 | 48 | 17 |
| $\mathrm{~V} \quad 3,5$ | 8 | 22 | 72 |  |
| $\mathrm{~V}_{\mathrm{sr}} \quad$ in $\mathrm{m} / \mathrm{s}$ | 0,6 | 0.40 | 0.46 | 4 |

Table 2.3: Aerodynamic parameters of differently sized flying machines as they occur in gliding flight, see also Fig. 5.3
by the way: $1 \mathrm{~m} / \mathrm{s}=3,6 \mathrm{~km} / \mathrm{h} \ldots$ you can derive that from the free space right here.

[^0] engines throttled back, except in the case of engine failure or lack of fuel [HL10].
H. Langenhagen
GTA Modellflug


Bild 3.1 The packaging view of the Climaxx. All the parts of the kit are CNC-milled and therefore worked accurately

- after the rather small flying models, we now dare to build a remote-control flying model
- it will be powered by an electric motor, but should also have good gliding characteristics
- we will hear more about the propulsion system later, for now it is about the
+ construction: it follows the principle of lightweight construction, i.e. the skin (foil) encloses a hollow body of low weight with the best possible strength. The fuselage carries the propulsion system and the receiver with control devices, some of which are also housed in the wings
$+\underline{\text { installation: this is done using the spar/rib/frame/skinning method. Although this method does not }}$ give the best contour accuracy (interesting for accuracy of the airfoil), it can be mastered by us. We will learn more about other construction methods later.


The box is open - can we match all the parts to the construction plan? Reading the drawing is required.


Marco and Paul start with the fuselage. Reinforcement is needed in the front part; we use casein glue (PVA) for these larger gluing areas. The glue can be spread well with a serrated spatula.


Karl and Antony build the wings. We thread the ribs on to the spars and position them over the building plan. The supervisor pins them to the spars, leading and trailing edge with superglue.

## Continuing construction Climaxx, wings

- the wing will consist of 2 centre sections and one "ear" each. We build these 4 wing parts over a blank copy, which is protected with a foil. When they are ready, one ear and one centre glued together to form half wings. Only for flight do you put the two halves together and and screw them to the fuselage.
- first, the long balsa ribs are threaded over the spars, inserted into the end rail and aligned then tacked with super glue. When building the centre sections, it is necessary to place the supplied $3 \times 20$ strip at the ends of the ribs; this gives the wing profile its planned camber. After gluing in the leading edge strip, also a carbon rod, we can insert the half ribs. Because the wing ears are trapezoidal, we must pay attention to the correct order of the ribs, which are of different lengths.
When the wing "ear" and the middle section have been glued together, we have to re-glue all the parts once again: the wood-wood joints with casein glue, the ribs-carbon spar joints with UHU-hard.
- The root ribs must be glued in slightly off plumb so that the desired dihedral is achieved when the wing parts are joined. That is why Mr Zimmermann glues them on himself with the help of an angle gauge.


## But why does the wing need such a thick tube as the main spar?

In figure 2.5 we had shown the forces acting on the model simplified as pointers, which all act at the centre of gravity. However, as already explained in Figure 2.7 the lift and drag ${ }^{2}$ forces are distributed over the entire wing surface due to the circulation flow around the wing along the span. The wing must absorb these forces and must not break even in the maximum load case.


Fig. 3.2 In normal flight the weight force acts on the wing. At the moment of interception from a dive a multiple of the (fuselage) weight force acts on the wing. Especially at the point where the wing is attached to the fuselage, a high bending moment occurs. The main spar must resist this, it must not break. In chapter 6 / Strength we learn how to calculate the necessary dimensioning.


Fig. 3.3 The main spar of a wing can be designed in different ways, here main and auxiliary spar tubes are made of Carbon fibre. Tubular spars add torsional rigidity.
A carbon fibre bar is also used on the wing leading edge, while the trailing edge is made of balsa. However, the leading and trailing edge do not contribute to the strength.

However, the wing is also loaded for twisting: At high airspeed, as can occur in dive, the point of attack of the lift shifts towards the trailing edge and tries to twist the trailing edge of the airfoil upwards. If the wing is "soft" in its structure, it follows this force effect to a certain degree, flaps back again, is bent upwards again, flaps back again ... a "flutter fall" is the inevitable consequence, from which the model can no longer be intercepted.

In many cases it has led to the destruction of a model when the pilot tried to bring it back from high altitude by nosediving. We achieve a safe descent from thermals without increasing speed by reducing lift and increasing drag, as is possible, for example, by extending ailerons upwards on both sides.

[^1]Continuing construction Climaxx, wing and fuselage


Now the "ear" and half of the centre section must be joined together. The two carbon moulded parts on the main spars ensure the exact dihedral angle and a firm connection. Therefore these are glued with 2-component-glue (Epoxy) by the exercise instructor.


After the fuselage has received its frame and servo mounts, We glue balsa panels cross-grained to the underside of the fuselage. It then looks like this $\uparrow$


To make it easier to handle later when assembling the wing at the airfield, we applied balsa blocks and plywood sheets ( 0.6 mm thick) to the wing root. When sanding the blocks, the nose area must often be checked for contour accuracy.

Which glue for which connection?
Depending on the type of bonding point, different glues are used for the shell, see Table 3.1. For example, tack bonding with superglue is practical because the steps can be carried out quickly one after the other due to the short curing time. If the adhesive is sprayed with the so-called "activator", the curing process is even accelerated. Another advantage of this adhesive is that it creeps between the bonding surfaces; the glue gaps
should be as thin as possible. But be careful: do not touch it with your fingers! In the mfcR club workshop, young people up to the age of 16 are only allowed to glue with casein glue! UHU-hart and casein glue take a little longer to cure, we use it to glue a plywood surface to a balsa slat, for example, or to re-glue the staples made with superglue. It is also very good for thickening strip-rib joints with glue plugs. 2-component adhesives are used for high-strength joints, whereby 5 -minute epoxy is usually suitable for our model applications. The following applies to all parts to be bonded: the surfaces must be clean, dry and free of oil, grease and dust!

|  | superglue | solvent glue | casein glue (PVA) | 2-component glue |
| :---: | :---: | :---: | :---: | :---: |
| trade name |  | UHU-hart, Duosan | Ponal, UHU Coll | 5Minute-Epoxy |
| glues well | wood $\Leftrightarrow$ wood <br> wood $\Leftrightarrow$ metal <br> wood $\Leftrightarrow$ plastics | wood $\Leftrightarrow$ wood wood $\Leftrightarrow$ plastics (Ribs on spar) | wood $\Leftrightarrow$ wood | wood $\Leftrightarrow$ wood <br> wood $\Leftrightarrow$ metal <br> wood $\Leftrightarrow$ plastics <br> plastics $\Leftrightarrow$ plastics |
| apply for | - lightly loaded bonds <br> - pinning parts <br> - difficult to access places <br> - time-critical bondings (airfield repairs) | - lightly loaded bonds <br> - small-area bondings | - loaded bonds <br> - large-area bondings | - highly stressed bondings - smaller bonding areas |
| processing time | 1s | 2Min | 10Min | 20Min |
| gluing time | 10s | 30Min | 1h |  |
| Further processing possible after | 1Min | 2h | 10h | 24h |
| Manufacturing Base | Cyanoacrylsäure | Cellulose-Nitrat, dissolved in ethyl ethanoate | PolyVinylACetat, distributed in aqueous solution |  |
| Hardens as a result of | chemical reaction (Polymerisation), start by moisture | Evaporation of the solvent | Evaporation of the water | chemical reaction (Polymerisation) after contact of resin and hardener |
| constancy | not moisture or temperature stable | insensitive to water | stable to moisture and temperature | stable to moisture and temperature |

Table 3.1 Model-making glue used so far; which is suitable for which connection?
Wing, fuselage and tailplane covering with heat shrink film


The ribs were initially attached to the spars with superglue. Now we glue these areas with UHU hard


First "ironing attempts" - here on the rudder blades.

## Tail unit construction

- the tailplane is designed as a V-tail, it consists of 2 surfaces (instead of 3 for the cruciform tailplane)
- first the rudders have to be attached to the tail planes by means of hinges ... this work

Mr. Zimmermann prefers to do it himself. Then we glue the two halves together using the moulded piece and then fit the complete tailplane into the fuselage.


The V-tail is glued together. It must now be fitted


into the recess at the rear of the fuselage.


Now we add plenty of white glue to the gluing surfaces and finally we have to remeasure and align,

so that it "fits" in every direction.


Here are some remaining glues at the wing root

### 4.0 We control club models



At the club's model airfield near Weißig, the remote-control instructors A. Lorenz (li) und H. Langenhagen await us with two different-sized powered gliders. We can send them via transmitters around in the sky. That makes fun!


At the beginning it is about flying at an altitude of aprox. 100 m along the field edge, flying routes each of which reverses direction with a $180^{\circ}$ turn.


With the smaller model we are allowed to try a landing. If we land too far away, we have to walk a little longer.

### 4.1 Longitudinal control



Fig. 4.1 You can imagine 3 axes that meet at the center of gravity. Using the control surfaces, we can turn the model around these axes in flight

A body in space always rotates around its center of gravity. This is also the case with our flight model. With the rudder deflections shown in Figure 4.1 it yaws about the vertical axis (rudder) and rolls around the longitudinal axis (aileron) into a left turn, and it nods downward around the lateral axis. Together this would result in a downward spiral trajectory. The three virtual axes intersect at the center of gravity cg of the model.
Already during the assembly of the balsa glider Jimmy, fig. 2.1, we promoted its position on the longitudinal axis by steel balls in the fuselage nose to the position indicated in the plan. In this model, it is located about 37 mm behind the leading edge of 1 wing.

In our remote control flights, we initiated the turns with the right stick and also released it at the end of the desired turn. The aileron on our club models is on this stick. Depending on your instructor's judgment, you may have been allowed to use the left stick to operate the elevator. While the effect of aileron and rudder is plausible in figure 4.1, we still need some knowledge about lift for the effect of elevator, figure 4.2.


Fig. 4.2 Interception arc during landing. With "pull elevator" the down force $F_{\text {AH }}$ causes a rotation around the center of gravity via the lever arm $\mathrm{r}_{\mathrm{H}}$. This continues until the angle of attack of the horizontal stabilizer $\alpha_{\text {anstr_ }}=0^{0}$ and thus its downforce has become zero. In the example shown, however, this causes the wing angle of attack to reach $\alpha_{\text {anstr_Fl }}=8^{0}$, the wing lift force increases, the model flies more slowly and touches down gently.

From the explanation for the development of the lift force $F_{L}$ Figure 2.6, it is known that its magnitude depends on the inflow velocity $\mathrm{v}_{\text {inflow }}$. However, the angle $\alpha$ at which the airfoil is approached also has an influence. Aerodynamicists gain knowledge about this in the wind tunnel, see Chapter 5 and here Figures 5.1a and b. Only this much should be said in advance: over a certain angle of attack range, the lift approximately follows a straight line; we will limit ourselves to this range here.

For the exemplary assumed angles of attack in figure 4.2, the top situation shows the model in stationary glide flight. With $\alpha_{\text {hs }}=0 \mathrm{deg}$ and because of the symmetrical airfoil, the horizontal tail does not generate any lift, see figure 5.2a), top characteristic curve. With the elevator angle in the middle situation, here


Fig. 4.3 A brief deflection at the elevator helps to bring the model back into normal flight
exemplarily $\alpha_{\text {hs }}-5 \mathrm{deg}$, this produces a down force, see figure 5.2 a , left straight line branch. This causes the model to rotate around the centre of gravity in such a way that it straightens up. As a result increases the wing angle of attack, the lift force on the wing increases up to the (practically impossible) limit case $F_{R}=F_{L}$, see figure 5.2b. The glide angle $\gamma$ decreases, thus also the propulsive force $\mathrm{F}_{\mathrm{T}}$ in figure 2.5 and the model lands at lower speed.

Figure 4.3, on the other hand, shows schematically how a critical situation, in which the longitudenal stability of the model can no longer do anything, can be remedied with a short depth rudder deflection. To do this, you have to catch the
right moment or the right flight path point: immediately after flying through the path minimum, you have to push down and the model resumes its glide path.

### 4.2 Curve control

For a properly steered turn, we operate all 3 rudders:

* with an aileron deflection we first "roll" the model around the longitudinal axis into an inclined position of our imagination;
*At the same time as the model takes up the bank angle, we add elevator "pull". This prevents the model from falling into a downward spiral. The greater the bank, the faster the model circles and the greater our elevator deflection must be so that it flies almost on a horizontal level;
*At the same time as the aileron deflection, we also add a slight rudder deflection, which supports the movement around the vertical axis, the yaw, according to the intended curve. Frequently experienced pilots often automate this process by adding a rudder deflection to the aileron deflection by programming their remote control system.

But before we fly full circles, we first practise corresponding partial circles: 90deg turns and 180deg turns. Already with the 90 deg turn, we will notice that the recovery of the turn, i.e. the roll back from the banked position to the horizontal wing position, makes the model suddenly rise. This is a result of speed increase: Despite all efforts to pull the elevator, we cannot prevent a model from sinking slightly in the curve and therefore accelerating in its orbital speed. ... and this excess speed is converted by each model into additional lift and thus into a short upward movement as soon as it returns to normal flight attitude with the wings horizontal. If you are not careful, this upward movement can even lead to a stall with subsequent "pumping", see figure 4.3, and it takes effort to return the model to normal flight. The remedy for this is again depth rudder deflection when the model comes out of the turn and starts to climb away..

However, explanations alone do not help. Quote Fritz Seidler for man-carrying flying [Seid62b]: "As in all fields, theory and practice must complement each other in flying. Neither the one who overemphasises theory nor the one who wants to do without it will achieve success."

## Rudder adjustments on the tail unit



Now the positions Servo zero and Rudder zero must be brought into line. To do this we provisionally hang the Bowden cable in the servo and cut it back to the necessary length.


On the tail side we bend a "Z" into the steel wire with special pliers and hook it into the rudder horn.


On the servo side, the steel wire is tinned and soldered into the sleeve of the rudder clip.


Then we set the appropriate length on the threaded rod of the rudder clip. The test shows: for the rudder deflections to be the same, the servo lever must be in the zero position at a different angle from 90 deg. We also cut off the unused levers so that they do not interfere with each other.

## Continuing with the fuselage



Now we glue the Bowden cables in several places in the fuselage and close it with balsa plates.


Although not planned in the construction plan, but nevertheless important: we reinforce the root area of the wing between the leading edge and the main spar with balsa blocks (not yet sanded to rib contour in the photos), between the main spar and the trailing edge with thin plywood planking. This gives more "grip-strength" for mounting and dismounting the model on the airfield.

Covering with iron-on foil



Now the wing is covered. This should form the airfoil shape given by the ribs along the entire span. The manufacturer has provided heat shrink film; it is coated on one side with a hot glue which melts and sticks when ironed on. First of all, the foil is only "tacked" to the end strip and ribs, then also to the leading edge strip .

... starting at the bottom. Then the "smoothing" (the stretching of the foil) takes place, while the iron is
slowly passed over the entire wing half.


The aileron is attached with adhesive tape.


The tail unit and the fuselage are also ironed


The temperature of the electric iron can be adjusted. First, we attach the film at about 120 deg celsius. then want to tighten larger areas, we can increase to 140 deg Celsius.


## Now we build a transport box

As soon as the model enters the "almost-ready" stage, a storage or transport box becomes necessary. It is made of corrugated cardboard. The dimensions are the result of considerations regarding the practical stacking of the model components.

The interior of the crate must be equipped in such a way, that the model is not damaged by rough transport procedures (falling over, dropping, etc.)


The box contains guide elements for threading the wing and fuselage made of impact-resistant EPP


The side walls of the fuselage tunnel are made of smooth cardboard so that the propeller does not get s when it is pulled out. The wings also get a (white) cover.


Closed fuselage tunnel, brackets for the wings. Now only the closing flap is missing

... and this is what the finished model looks like now!!!

### 5.0 What we can work out with the air forces

5.1 Calculation with coefficients

In Fig. 2.5 we learned about the forces acting on the flight model in stationary, i.e. in unaccelerated glide flight. Using a simple estimation for the aerodynamic quality E from flight altitude and flight distance, see Table 2.2, we were even able to make statements about the magnitude of the lift force $\mathrm{F}_{\mathrm{L}}$ $\approx 0.64 \mathrm{~N}$ and drag force $\mathrm{F}_{\mathrm{D}} \approx 0.11 \mathrm{~N}$ acting on the model Jimmy. We can see that the drag force with $\mathrm{F}_{\mathrm{D}}$ $=\mathrm{F}_{\mathrm{L}} \mathrm{E} \approx 0.64 \mathrm{~N} / 6$ is much smaller than the lift force. This is just as well, because if this value were larger, E would be smaller and our Jimmy would not have flown so far at all.
But can the quantities $F_{L}, F_{D}$ be obtained without flight measurements? Otto LILIENTHAL already made experiments with a circular run, Figure 5.1 a. Later, wind tunnels were built in which airfoil models were installed and the forces lift and drag could be measured depending on the angle of attack $\alpha$, Figure 5.1 b.


FIG. 5.1a: Roundabout of LILIENTHAL in 1873. The weights help the rotational movement; depending on the angle of attack $\alpha$, the resulting lift force $F_{A}$ can be measured with the help of the centrally located balance [Böl90].


FIG. 5.1b: Air force measurement on the airfoil model mounted in the wind tunnel [SLZ77]. The wing position is maintained here by control, the measured value is generated via the forcedeflection which is proportional of the balances (compensation principle).

However, it was soon recognized that the measurement results depend not only on the angle of attack and the airfoil shape flowing around the surface, but also on the surface size $\mathrm{S}_{\mathrm{w}}$ and the inflow velocity v . For this reason, the dimensionless coefficients $\mathrm{c}_{\mathrm{d}}$ for the lift coefficient and $c_{d}$ for the drag coefficient were introduced, which the wind tunnel engineers determine from the force measurements. Conversely, every user can now calculate the forces to be expected for his aircraft or model aircraft on the basis of the measured coefficients [BB14].

$$
\begin{equation*}
F_{L}=\frac{\rho}{2} \cdot v^{2} \cdot S \cdot C_{l} \tag{5.1}
\end{equation*}
$$

und

$$
\begin{equation*}
F_{D}=\frac{\rho}{2} \cdot v^{2} \cdot S \cdot c_{d} \tag{5.2}
\end{equation*}
$$

In this
v : Air velocity (m/s)
$\rho:$ Air density, $\rho_{\text {air }} \approx 1.23 \mathrm{~kg} / \mathrm{m}^{3}$ bei $\vartheta=15^{\circ} \mathrm{C}$
S : Projection of the washed around area $\left(\mathrm{m}^{2}\right)$
$c_{1}$ : Lift coefficient of the surfae
$c_{d}$ : Drag coefficient of the surfae
Results of such measurements for the coefficients on two airfoils are shown in Figures 5.2-a) and b). The difference between the two airfoils is the camber of the centre line: the NACA0012 has the camber $\mathrm{f}=0 \%$, the CLARK Y those of $\mathrm{f}=3.6 \%$. Both are the same thickness, $\mathrm{d}=12 \%$.


FIG. 5.2: The measured values of two airfoils in comparison [A80]
a) symmetrical: NACA $0012, d / \mu_{\mu}=0,12 ; f / \mu_{\mu}=0$
b) cambered: Clark $Y, d / l_{\mu}=0,12 ; f / I_{\mu}=0,036$ We first look at diagram line_2. The course of the lift coefficient $c_{1}$ over the angle of attack $\alpha$ resembles a straight line within a certain angle of attack range; better in the case of the symmetrical
airfoil, somewhat worse in the case of the curved (also called: asymmetrical) airfoil. With both, however, $c_{1}$ does not grow arbitrarily far. Above a certain angular value, it even becomes smaller again.
It is also noticeable that the characteristic curve of the cambered airfoil is shifted towards higher $\mathrm{c}_{1}$ values; here, higher lift coefficients $c_{1 \_m a x}$ can therefore be achieved with larger angles of attack than with a symmetrical profile.

The course of the drag coefficient, diagram line_3 over $\alpha$, rather resembles a quadratic function. In the case of the symmetrical profile it reaches its minimum at $\alpha=0 \mathrm{deg}$, in the case of the cambered airfoil at a somewhat more positive angle of attack, $\alpha=2 \mathrm{deg}$. In both cases, however, the drag coefficient is small with $\mathrm{c}_{\mathrm{d}} \leq 0.03$ compared to $\mathrm{c}_{1 \_\max } \approx 1$ in the angle of greatest lift.

The unification of lift and drag coefficient diagrams into a polar diagram with the angle of attack $\alpha$ as a running variable (line_4) also goes back to LILIENTHAL. We will show later how useful this representation is.
But what can we do with such diagram values? Let us first look at some examples of the maximum lift coefficient $\mathrm{c}_{1}$ max.

Example 5.1 (see also pictures 5.3, 5.7 and 6.18)
a) The wing area of the slowflyer RL1 built of foam polystyrene is $\mathrm{S}_{\mathrm{f}}=17,8 \mathrm{dm}^{2}$, its mass is $\mathrm{m}=140 \mathrm{~g}$. It flies (in cruise flight) at a speed of $v \approx 5 \mathrm{~m} / \mathrm{s}$ fast. With what lift coefficient $\mathrm{c}_{1}$ does the model fly? From Eq(5.1) it follows with $\mathrm{F}_{\mathrm{L}}=\mathrm{m} * \mathrm{~g}$

$$
c_{l}=\frac{m \cdot g}{\frac{\rho}{2} \cdot v^{2} \cdot S_{f}}
$$

One calculates $\mathrm{c}_{1}=0.14 \mathrm{~kg}^{*} 9.81 * \mathrm{~m}^{*} \mathrm{~s}^{-2} /\left[(\rho / 2) * \mathrm{v}^{2} * \mathrm{~S}_{\mathrm{F}}\right]=1,37 \mathrm{kgm}^{*} \mathrm{~s}^{-2} /\left[0,5 * 1,23 \mathrm{~kg}^{*} \mathrm{~m}^{-3} *\left(5 \mathrm{~m}^{*} \mathrm{~s}^{-1}\right)^{2 *} 0,178 \mathrm{~m}^{2}\right]=0,5$
b) The performance glider model RS3 developed in the Modellflugclub Rossendorf e.V. has the wing area
$\mathrm{S}_{\mathrm{f}}=68 \mathrm{dm} *$ and the mass $\mathrm{m}=2,4 \mathrm{~kg}$. During the test flights the following values were determined by on-board measurements [JW04]: $\mathrm{v}=9 \mathrm{~m} / \mathrm{s}, \mathrm{E}=22$. With which lift coefficient $\mathrm{c}_{1}$ did the model fly?
One calculates $\mathrm{c}_{1}=2,4 \mathrm{~kg} * 9.81 * \mathrm{~m}^{*} \mathrm{~s}^{-2} /\left[(\rho / 2) * \mathrm{v}^{2} * \mathrm{~S}_{\mathrm{f}}\right]=23,54 \mathrm{kgm}^{*} \mathrm{~s}^{-2} /\left[0,5 * 1,23 \mathrm{~kg}^{*} \mathrm{~m}^{-3} *\left(9 \mathrm{~m}^{*} \mathrm{~s}^{-1}\right)^{2} * 0,68 \mathrm{~m}^{2}\right]=0,69$
Although we do not (yet) know the $c_{1}$ curves of the installed airfoil in cases $a, b$, the $c_{1}$ values seem plausible when compared with the diagrams Figure 5.2.
c) According to the data sheet [Jant78] the performance sailplane JANTAR 2b, first flight 1977, has the wing area $\mathrm{S}_{\mathrm{f}}=14.25 \mathrm{~m}^{2}$. The specific mass of the wing can be adjusted from $\mathrm{B}=30.3 \mathrm{~kg} / \mathrm{m}^{2}$ to $45.5 \mathrm{~kg} / \mathrm{m}^{2}$ via the water ballast which can be carried along. The minimum permitted (stall) speed for Bmax $=45.5 \mathrm{~kg} / \mathrm{m}^{2}$ is vmin $=$ $80 \mathrm{~km} / \mathrm{h}=22.2 \mathrm{~m} / \mathrm{s}$. What is the lift coefficient $\mathrm{c}_{\mathrm{l} \_ \text {max }}$ at this speed?

$$
c_{l_{-} \max }=B_{\max } \cdot \frac{g}{\frac{\rho}{2} \cdot v^{2}}
$$

Mit $\mathrm{B}_{\max }=45,5 \mathrm{~kg} / \mathrm{m}^{2}$ und $\mathrm{g} /\left[(\rho / 2)^{*} \mathrm{v}^{2}\right]=9,81 * \mathrm{~m}^{*} \mathrm{~s}^{-2} /\left[0,5 * 1,23 * \mathrm{~kg}^{*} \mathrm{~m}^{-3} *\left(22,2 * \mathrm{~m}^{*} \mathrm{~s}^{-1}\right)^{2}\right]=0,052$ wird $\mathrm{c}_{\text {amax }}=1,47$. However, this lift coefficient is already slightly above the lift curves shown in diagram line_2.
d) At ICAO standard atmosphere: $\mathrm{h}=$ sea level; $\theta=15 \mathrm{degC} ; \mathrm{p}=1013.25 \mathrm{hPa} ; \rightarrow \rho=1.225 \mathrm{~kg} / \mathrm{m}^{3}$ the Airbus A320 needs a speed of $v_{\text {lift }}=270 \mathrm{~km} / \mathrm{h}=75 \mathrm{~m} / \mathrm{s}$ for takeoff with maximum mass $\mathrm{m}_{\max }=77000 \mathrm{~kg}$. Its wing area is $\mathrm{S}_{\mathrm{f}}=122.6 \mathrm{~m}^{2}$ [A320_a] to [A320_c]. What lift coefficient $\mathrm{c}_{1 \_\max }$ must the airfoil provide during lift-off, and what lift coefficient $\mathrm{c}_{\mathrm{a}_{2} \min }$ is required during cruise flight at 10 km altitude $\left(\rho=0.41 \mathrm{~kg} / \mathrm{m}^{3}\right)$ with $\mathrm{V}_{\text {cruise }}=900 \mathrm{~km} / \mathrm{h}=$ $250 \mathrm{~m} / \mathrm{s}$ ?
The calculation results in $\mathrm{c}_{1 \_\max }=1.78$ for lift-off; $\mathrm{c}_{1}$ cruise $=0.48$ for cruise flight.

The examples show that the maximum achievable lift coefficients of $\mathrm{c}_{1 \_} \max =1.47$ (Jantar) and 1.78 (A320) far exceed the maximum values shown in Fig. 5.2a), b). In practice, this is achieved by the fact that the camber of the airfoil and, in some cases, its wing area can be increased during flight, Fig 5.3


## Airfoil geometry


classic rigid airfoil (Gö 417a) with high camber $f=5,8 \%$
RS3



Flap angle $\eta=+5^{0}$, soaring in thermals


Flap angle $\eta=-5^{0}$, fast flight

Glider Jantar 2b


Flap angle $\eta=+16^{0}$, soaring in thermals


Slat and Flap retracted, cruise flight


Fig. 5.3: Variable airfoil geometry and its effect on the lift coefficient

In specific cases, the following procedures are used for this purpose:

- a flap adjustable in its angle at the end of the airfoil causes an increase in camber or a camber reduction, see RS3 and JANTAR 2b;
- A flap at the end of the airfoil that can be extended downward and backward (fowler flap) leads to an increase in camber and a lengthening of the Airfoil chord, it also increases the lift area, see A320;
- the retractable leading edge flap (slat) prevents stall on the wing at high angles of attack by supplying high-energy air, see A320.

In the literature [Neu67] it is stated that lift coefficients up to $\mathrm{c}_{1 \_\max }=3.6$ can be achieved by combining slats and Fowler flaps.
In the field of model flight, however, only deflectable flaps have been used so far; there are also measurement results for them [A80], [Se89]. The flaps are used in soaring flight models (class F3J, example model type RS3), for which slow flight with the smallest possible curve radii is desired when circling in thermals. In the "royal class" F3B of RC gliding, on the other hand, a model must also complete the disciplines of cross-country flight and speed flight in addition to thermal flight. We have already seen from the example of A320 cruise flight that for high speeds only a small lift coefficient ca (i.e. a small camber) is necessary, because in the product equation Eq. 5.1 for FA then the speed v with its quadratic influence dominates. This is the reason why in F3B models "fast" airfoils (those with low camber) are built in as far as possible, which can then be induced to fly slowly by a positive camber. It is possible that slats are also used in scale modeling (where lifelike reproduction is important); however, nothing is known about measurement results in the Reynolds number range of model flying.

### 5.2 The stationary flight states

Let's now take a closer look at the forces acting on the model/airplane in the flight conditions glide, level flight, climb. We will see that we can do even more with the lift and drag coefficients $\mathrm{c}_{1}, \mathrm{c}_{\mathrm{d}}$.

## 5.2-1 Gleitflug



Fig 5.4: Forces and speeds in unaccelerated gliding flight. The forces act at the centre of gravity of the model. The weight force $\mathrm{Fw}_{\mathrm{w}}$ can be decomposed into the components $\mathrm{F}_{\mathrm{w} 1}=\mathrm{Fw}^{*} \cos (\gamma)$ and $\mathrm{Fw}_{2}=\mathrm{Fw}^{*} \sin (\gamma)$. The path velocity v can be decomposed into $\mathrm{v}_{\mathrm{sr}}=\mathrm{v}^{*} \sin (\gamma)$ and $\mathrm{v}_{\mathrm{x}}=\mathrm{v}^{*} \cos (\gamma)$

The aim of our considerations is to obtain information on the path speed $v$ and the sink rate $\mathrm{v}_{\mathrm{sr}}$ of the model. For this purpose, we assume that the forces act at the centre of gravity.
A model without propulsion flies from an altitude $\Delta \mathrm{h}$ with a glide angle $\gamma>0 \mathrm{deg}$, i.e. with a constant loss of altitude over a distance $\Delta \mathrm{s}$, Figure 5.4. It is driven by the thrust force $\mathrm{F}_{\mathrm{W} 2}=\mathrm{F}_{\mathrm{W}} * \sin (\gamma)$, a component of the omnipresent weight force $\mathrm{F}_{\mathrm{W}}$. We can already see from this that a glide angle $\gamma=0$ deg is not possible in propulsionless flight: because $\sin (\gamma=0)=0$, the propulsive force would also be missing due to $\mathrm{F}_{\mathrm{W} 2}=\mathrm{F}_{\mathrm{W}} * \sin (\gamma=0)=0$.

The forces are in equilibrium, $\mathrm{F}_{\mathrm{L}}=\mathrm{F}_{\mathrm{W} 1}, \mathrm{~F}_{\mathrm{w}}=\mathrm{F}_{\mathrm{W} 2}, \mathrm{~F}_{\mathrm{R}}=\mathrm{F}_{\mathrm{w}}$. For the upper triangle of forces we apply the Pythagorean theorem:

$$
F_{R}=F_{W}=\sqrt{F_{L}^{2}+F_{D}^{2}}
$$

If we use the relationships for lift and drag force according to equations (5.1) and (5.2), we can calculate the speed at which the model flies on its inclined path in unaccelerated glide flight:

$$
\begin{equation*}
v=\sqrt{\frac{m \cdot g}{S_{w} \cdot \frac{\rho}{2}} \cdot \frac{1}{\sqrt{c_{l}{ }^{2}+c_{d}{ }^{2}}}} \tag{5.3}
\end{equation*}
$$

But this also gives us an explanation for the "correct" use of force during the hand launch of our balsa glider JIMMY via a formula: due to its parameters (mass m , wing area $\mathrm{S}_{\mathrm{w}}$ as well as lift and drag parameters $c_{l}, c_{d}$ ) there is a certain flight path speed $v$ for each model with which it performs the unaccelerated steady glide flight. Values deviating from this, as they occur during manual take-off when too little or too much power is applied, lead either to a strong descent (curve bin fig 2.3) or to a climb and subsequent "festoon flight", (curve a in fig 2.3). In section 4.1/Longitudinal control we have already learned how we should react to such flight phases in RC flight, see figure 4.3.
But there is another insight: the airspeed in gliding flight is not determined by the model mass alone, but rather by the specific wing mass $\mathrm{m} / \mathrm{S}_{\mathrm{w}}$.
From the above calculation approach, the rate of descent $\mathrm{v}_{\mathrm{sr}}$ can also be given:

$$
\begin{equation*}
v_{s r}=\sqrt{\frac{m \cdot g}{S_{w} \cdot \frac{\rho}{2}} \cdot \frac{c_{d}{ }^{2}}{\sqrt{\left(c_{l}{ }^{2}+{\left.c_{d}{ }^{2}\right)^{3}}^{2}\right.}}} \tag{5.4}
\end{equation*}
$$

We have heard the term glide ratio or aerodynamic quality before (Tabless 2.2 und 2.3):

$$
\begin{equation*}
\frac{1}{\varepsilon}=E=\frac{h}{s}=\frac{v_{x}}{v_{s r}} \tag{5.5a}
\end{equation*}
$$

or for small glide angles $\gamma$ :

$$
\begin{equation*}
\frac{1}{\varepsilon}=E=\frac{h}{s} \approx \frac{v}{v_{s r}} \tag{5.5b}
\end{equation*}
$$

For the numerical estimation of the velocities, we go back to the coefficient curves in Figures 5.2 a) and $b$ ). We had noticed that the lift coefficient $c_{l}(\alpha)$ within a certain angle of attack range resembles a straight line, whereas the drag coefficient $\mathrm{c}_{\mathrm{d}}(\alpha)$ rather resembles a quadratic function. We want to make use of this and calculate the velocities approximately. As an approximation function ${ }^{3}$ for the lift

[^2]coefficient we write
and for those of the drag coefficient
\[

$$
\begin{equation*}
c_{d}(\alpha)=p \cdot\left(\alpha-\alpha_{1}\right)^{2}+c_{d 0} \tag{5.6b}
\end{equation*}
$$

\]

Here we take from the courses for the CLARK_Y: $q=0,092 /$ deg and $a_{0}=-2,5^{0}, p=0,65$ und $\alpha_{1}=2^{0}$, $c_{\mathrm{d} 0}=0,008$.

However, it is not only the airfoil of the wing that creates drag. Rather, we must consider that as a result of the pressure difference between the upper side (negative pressure) and the lower side (positive pressure), a pressure equalisation takes place at the wing tips, compare Figure 2.7. This happens with the formation of a draggy vortex, a further drag is induced. The edge flow forms a vortex plait behind the wing tip; Figure 5.5 shows how this vortex ${ }^{4}$ plait is to be interpreted as a continuation of the airfoil flow.


Fig 5.5: Vortex system of aerofoil in flight. Ludwig PRANDTL speaks of a horseshoe vertebra [Pra35]. The bundled vortex provides the lift.


Fig 5.6: The German commercial aircraft Heinkel
He 70 has an elliptical wing plan.
Maiden flight of the prototype:
December 1933, v=370km/h. [KN77]

This induced drag is smaller the larger the span $b$ is in relation to the wing chord length $1 \mu$. The ratio is called wing aspect ratio A. Depending on the known size, see Figure 5.7, it is calculated as follows

$$
\begin{equation*}
A=\frac{b}{l_{\mu}}=\frac{S_{w}}{l_{\mu}^{2}}=\frac{b^{2}}{S_{w}} \tag{5.7}
\end{equation*}
$$

The coefficient of the induced resistance $\mathrm{c}_{\mathrm{di}}$ obeys the relation

$$
\begin{equation*}
c_{d i}=\frac{c_{l}^{2}}{\pi \cdot A} \tag{5.8}
\end{equation*}
$$

[^3]Aircraft
Slowflyer
RL1


## Aspect ratio A

$\mathrm{b}=8,7 \mathrm{dm}$
$S_{w}=16.34 \mathrm{dm}^{2}$
$A=b / l_{\mu}=870 / 190$
$\mathrm{A}=\mathbf{4 , 6}$
b $=34 \mathrm{dm}$
$S_{w}=68 \mathrm{dm}^{2}$
$A=b^{2} / S_{w}$
$A=17$

$$
\begin{aligned}
b & =20,5 \mathrm{~m} \\
\mathrm{~S}_{\mathrm{w}} & =14,25 \mathrm{~m}^{2} \\
\mathbf{A} & =\mathbf{2 9 , 4}
\end{aligned}
$$

$$
\begin{aligned}
b & =33,9 m \\
S_{w} & =122,6 m^{2} \\
\mathbf{A} & =9,4
\end{aligned}
$$

Fig 5.7: Wing aspect ratios A for different wing planforms. For gliders, large values are used in the interest of slow thermal circling; for powered aircraft, aspect ratio is designed according to economic considerations (range, speed in commercial flight) or according to manoeuvrability (aerobatics).
where the influence of the aspect ratio A becomes clear. Strictly speaking, this formula only applies to a wing plan with an elliptical shape, see Figure 5.6 ; this produces the lowest induced drag of all geometries. However, we continue to use it as an approximation.
Now, in addition to the wing, the fuselage/tail boom and the tail unit also belong to the aircraft/airplane model; both components are washed around by the air during flight and generate resistance. In the coefficient notation we take this into account with the so-called harmful resistance $\mathrm{c}_{\mathrm{dh}}$ (although actually any kind of resistance is harmful). Thus the relationship for the total drag coefficient is now

$$
\begin{equation*}
c_{d}(\alpha)=p \cdot\left(\alpha-\alpha_{1}\right)^{2}+c_{d 0}+\frac{c_{l}^{2}}{\pi \cdot A}+c_{d h} \tag{5.6c}
\end{equation*}
$$

Now we can calculate the course of the airspeed and sink rate depending on the angle of attack according to equations (5.3), (5.4). As an example we use the mentioned glider model RS3 with the values $S_{w}=68 \mathrm{dm}^{2}, m=2.4 \mathrm{~kg}, A=17$ and $\mathrm{c}_{\mathrm{ds}}=0.015$. We vary the wing angle of attack in the range $0 \operatorname{deg} \alpha \leq 10 \operatorname{deg}$ by trimming the elevator, thus producing different glide angles $\gamma$. The results are shown in Figures 5.8a) to d).


Fig 5.8: Speed ranges of a glider model in gliding flight. a) Path velocity v b) Sink rate $\mathrm{v}_{\text {sr. }}$. The glide angle $\gamma$ varies only slightly: from $\gamma=2$ deg (slow flight) to $\gamma=5 \mathrm{deg}$ (fast flight).


Fig 5.8: c) Velocity polar $v_{s r}(v)$ and derived from this $d$ ) the course of aerodynamic quality $E(v)=1 / \varepsilon(v)$ The tangent point lies at the maximum $E_{\max }=E(\alpha=6 \mathrm{deg})$

We see that changing the angle of attack changes the orbital velocity of the model in the range $16 \mathrm{~m} / \mathrm{s} \geq \mathrm{v}$ $\geq 7 \mathrm{~m} / \mathrm{s}$, while the corresponding sink rate varies in the range $1.6 \mathrm{~m} / \mathrm{s} \geq \mathrm{v}_{\mathrm{sr}} \geq 0.29 \mathrm{~m} / \mathrm{s}$. Almost more interesting than the numerical values themselves, however, are the curves: one can see that the path velocity $\mathrm{v}(\alpha)$ could very well be reduced by an even higher angle of attack (mathematically equivalent to a higher lift coefficient), but that the rate of descent $\mathrm{v}_{\mathrm{sr}}(\alpha)$ is heading towards a stationary final value.

Furthermore, for the evaluation of gliders it has become common to show the (negative) sink rate $\mathrm{v}_{\mathrm{sr}}(\alpha)$ above the airspeed $v(\alpha)$, Figure 5.8c. From this curve, called the speed polar curve, one not only recognises the minimum permissible path speed $\mathrm{v}_{\text {min }}$ and the corresponding minimum sink rate $\mathrm{v}_{\mathrm{sr} \_}$min. Rather, the slope of a straight line between the origin of the coordinates and the tangent point also allows a statement to be made about the best possible glide ratio $\varepsilon$ or aerodynamic quality $\mathrm{E}=1 / \varepsilon$. It is also interesting to note that $\mathrm{E}_{\max }$ with $\alpha=6 \mathrm{deg}$ is achieved at a different angle of attack than the minimum sink rate $\mathrm{v}_{\text {sr_min }}$ at $\alpha=10 \mathrm{deg}$. This plays a role in competition flights of gliders and glider models: for slow circling in thermals the lower speeds are needed, while for cross-country flight the best glide ratio is required.

The above calculation of characteristic values - and also the following ones - are intended to describe the basic procedure. However, due to the approximations, absolute values can only be determined with a certain tolerance; for example, the aerodynamic quality with $\mathrm{E}_{\max }=25$ deviates by $\approx 10 \%$ from the measured value $\mathrm{E}=22$, cf. example 5.1 b ). On the one hand, more accurate calculations require the use of airfoil polars measured in the Reynolds number range of the flight model, see chapter 7, on the other hand, measured values also have their tolerances, see section 8.5.
Figure 5.9 shows the calculated individual resistances of an example glider model for the flight modes slow flight a) and fast flight b). We see that in slow flight the induced drag dominates, in fast flight the airfoil drag.


Fig. 5.9: Drag forces on a glider model a) with high, b) with low lift coefficient

## 5.2-2 Horizontal flight

Let us now look at stationary horizontal flight. This is caused by a thrust force $\mathrm{F}_{\mathrm{T}}$ acting on the model due to an installed propulsion system (propeller, jet engine), Figure 5.10. Here, too, the forces are in equilibrium, $\mathrm{F}_{\mathrm{L}}=\mathrm{F}_{\mathrm{W}}, \mathrm{F}_{\mathrm{T}}=\mathrm{F}_{\mathrm{D}}$. With knowledge of Eq.(5.1), (5.2) we can write


Fig 5.10: Forces in stationary horizontal flight

$$
\begin{equation*}
F_{T}(\alpha)=m \cdot g \cdot \frac{c_{d}(\alpha)}{c_{l}(\alpha)} \tag{5.9}
\end{equation*}
$$

$$
\begin{gathered}
m \cdot g=\frac{\rho}{2} \cdot v^{2} \cdot S_{w} \cdot c_{l} \\
F_{T}=\frac{\rho}{2} \cdot v^{2} \cdot S_{w} \cdot c_{d}
\end{gathered}
$$

These two equations are fulfilled for every speed. We can therefore solve for $\mathrm{v}^{2}$ and equate it to determine the thrust force:
and the velocity to

$$
\begin{equation*}
v(\alpha)=\sqrt{\frac{m \cdot g}{S_{w} \cdot \frac{\rho}{2} \cdot \frac{1}{c_{l}(\alpha)}}} \tag{5.10}
\end{equation*}
$$

In the figures Fig. 5.11 a) and b) these relations are evaluated for our example model RS3, which now has a higher mass of $\mathrm{m}=3 \mathrm{~kg}$ due to the motorisation. The results are shown on a semi-logarithmic scale in order to better recognise the respective minimum values; for the thrust force, for example, this is $\mathrm{F}_{\mathrm{T}}=1.2 \mathrm{~N}$ for $\alpha=7 \mathrm{deg}$.
It should be emphasised once again that the speed range of the model in horizontal flight is being investigated here. This results from the fact that for a given angle of attack range $\alpha_{0}<\alpha \leq 10 \mathrm{deg}$, both the thrust force $\mathrm{F}_{\mathrm{T}}$ and the speed v are determined which are necessary to keep the model in horizontal flight, where the lift force is therefore always equal to the weight force $\mathrm{F}_{\mathrm{L}}=\mathrm{F}_{\mathrm{w}}$. Of course, the thrust force curve $\mathrm{F}_{\mathrm{T}}$ conforms to the resistance force curve $\mathrm{F}_{\mathrm{D}}$, because only the latter must be in equilibrium with the thrust force for stationary horizontal flight.


FIG. 5.11a) speed range and b) required thrust of a model for horizontal flight.
In our calculation example, according to figure 5.11, the speed for this is in the range $\mathrm{v}_{\text {min }}=7.8 \mathrm{~m} / \mathrm{s}$ to $\mathrm{v}_{\text {max }}=87 \mathrm{~m} / \mathrm{s}$, for which a thrust of $\mathrm{F}_{\mathrm{T}_{-} \min }=1.2 \mathrm{~N}$ to $\mathrm{F}_{\mathrm{T}_{-} \max }=86 \mathrm{~N}$ must be available.

However, it should be noted that the maximum speed is a calculated value which cannot be realised in practice - another variable, the so-called torque around an imaginary axis in the aerofoil, wants to twist (torsion) the aerofoil counter-clockwise at small angles of attack and thus leads to oscillations (flutter) of the aerofoil, which can destroy it.

## 5.2-3 Climb flight

A first, very descriptive consideration [Br80], Figure 5.12, results from the following consideration: if a flying model flies without propulsion, it sinks in the time $\Delta \mathrm{t}$ by the height $\Delta \mathrm{h} 1$. The sink rate results from

$$
v_{s r}=\frac{\Delta h 1}{\Delta t}
$$

If the motor is switched on, the airscrew generates the forward thrust force $\mathrm{F}_{\text {th }}$. If this is equal to the drag force, $F_{T}=F_{D}$, then the model flies horizontally, see Figure 5.10. If the propulsive force is increased, $\mathrm{F}_{\mathrm{th}}>\mathrm{F}_{\mathrm{d}}$, then the model moves upwards at the rate of climb:

$$
v_{c r}=\frac{\Delta h 2}{\Delta t}
$$



Fig12: In the case of small climbing angles, the required drive power can be determined from the lifting work.
 都

The propulsion system thus performs a lifting work in the climb with the force component weight force $\mathrm{F}_{\mathrm{W}}=$ mass*g and the displacement components Altitude difference $\Delta \mathrm{h}=\Delta \mathrm{h} 1+\Delta \mathrm{h} 2$ : Altitude difference $\Delta \mathrm{h}=\Delta \mathrm{h} 1+\Delta \mathrm{h} 2$ :

$$
W_{c r}=F_{W} *(\Delta h 1+\Delta h 2)
$$

From the lift work $\mathrm{W}_{\mathrm{cr}}$ and the time interval $\Delta \mathrm{t}$ we can determine the desired output power $\mathrm{P}_{\text {out }}$ for the climb:

$$
\begin{align*}
& P_{\text {out }}=\frac{W_{c r}}{\Delta t} \\
& P_{\text {out }}=F_{W} \cdot\left(\frac{\Delta h 1+\Delta h 2}{\Delta t}\right) \\
& P_{\text {out }}=m \cdot g\left(v_{s r}+v_{c r}\right) \tag{5.11}
\end{align*}
$$

This power must be delivered by the propeller to propel the model into the sky at a rate of climb $\mathrm{v}_{\mathrm{cr}}$. The sink rate $\mathrm{v}_{\text {sr }}$ must be known for this - either calculated according to $\mathrm{Eq}(5.4)$ or available as a measured value. However, we will soon see that this relationship only provides a sufficiently accurate result for small angles of climb $\gamma \leq 10$.

For calculation with arbitrary angles of climb we consider the forces according to figure 5.13.


Fig 5.13: Forces in unaccelerated climb. The weight force $F G$ is decomposed into the components $F_{G 1}=F_{A}=F_{G}{ }^{*} \cos \gamma, F_{G 2}=F_{G}{ }^{*} \sin \gamma$. Damit gilt $F_{S}=F_{G}{ }^{*} \sin \gamma+F w$. Außerdem gilt $v_{s t}=v^{*} \sin (\gamma)$

Compared to horizontal flight (compare figure 5.10), the climb results from a greater thrust force $\mathrm{F}_{\text {th }}$ acting on the model/aircraft than is necessary to compensate for the drag force $\mathrm{F}_{\text {th }}$. According to figure 5.13 the following forces are in equilibrium: $\mathrm{F}_{\mathrm{S}}=\mathrm{F}_{\mathrm{G} 2}+\mathrm{F}_{\mathrm{W}}, \mathrm{F}_{\mathrm{A}}=\mathrm{F}_{\mathrm{G} 1}=\mathrm{F}_{\mathrm{G}} * \cos \gamma, \mathrm{~F}_{\mathrm{G} 2}=\mathrm{F}_{\mathrm{G}} * \sin \gamma$

Again using the basic equations (5.1), (5.2) we can write

$$
\begin{equation*}
F_{\mathrm{W}} \cdot \cos (\gamma)=\frac{\rho}{2} \cdot v^{2} \cdot S_{f} \cdot c_{a} \tag{5.12a}
\end{equation*}
$$

and for the shear force FS the sum of the weight force component FG2 and the resistance force FW according to (5.2) applies:

$$
\begin{equation*}
F_{T}=F_{W} \cdot \sin (\gamma)+\frac{\rho}{2} \cdot v^{2} \cdot S \cdot c_{d} \tag{5.12b}
\end{equation*}
$$

By solving both equations for $v^{2}$ and equating, we obtain for the thrust force

$$
F_{T}(\alpha)=F_{W} \cdot\left[\frac{c_{d}(\alpha)}{c_{l}(\alpha)} \cdot \cos (\gamma)+\sin (\gamma)\right]
$$

But since also applies

$$
v_{c r}=v \cdot \sin (\gamma)
$$

the necessary thrust force is

$$
F_{T}(\alpha)=F_{W} \cdot\left[\frac{c_{d}(\alpha)}{c_{j}(\alpha)} \cdot \cos (\gamma)+\frac{v_{c r}}{v}\right]
$$

and the required power

$$
P_{\text {out }}=F_{T}(\alpha) \cdot v=F_{W} \cdot\left[\frac{c_{d}(\alpha)}{c_{l}(\alpha)} \cdot v \cdot \cos (\gamma)+v_{c r}\right]
$$

For small gradient angles $\gamma \leq 100$, the value of $\cos (\gamma) \geq 0.99 \approx 1$ remains; moreover, the following applies

$$
\frac{c_{w}(\alpha)}{c_{a}(\alpha)} \cdot v \approx v_{s i}
$$

and thus in this case also

$$
P_{o u t} \approx F_{W} \cdot\left(v_{s r}+v_{c r)}\right)
$$

This confirms our equation (5.11) obtained from the first observation.
For the larger angular range $0 \mathrm{deg}<\gamma<89 \mathrm{deg}$ we calculate the velocities from (5.12a) according to

$$
\begin{equation*}
v(\alpha)=\sqrt{\frac{2 \cdot F_{W} \cdot \cos (\gamma)}{\rho \cdot S_{w} \cdot c_{a}(\alpha)}} \tag{5.13}
\end{equation*}
$$

and because of $v_{c r}=v \cdot \sin (\gamma)$

$$
\begin{equation*}
v_{c r}(\alpha)=\sin (\gamma) \cdot \sqrt{\frac{2 \cdot F_{W} \cdot \cos (\gamma)}{\rho \cdot S_{w} \cdot c_{a}(\alpha)}} \tag{5.14}
\end{equation*}
$$

But how do we determine the angle of climb $\gamma$ and how can the thrust force $\mathrm{F}_{\mathrm{T}}$ the included in the calculation? A closed solution is not possible here, rather iteration must help: For a chosen initial value $\gamma_{0}$ and the desired angle of attack $\alpha$ we calculate the path velocity $v\left(\alpha, \gamma_{0}\right)$ according to Eq. (5.13) and then the drag force $\mathrm{F}_{\mathrm{T}}(\alpha, \gamma 0)$ with Eq. (5.2) and check with the help of the relation, cf. Fig. 5.13,

$$
\begin{equation*}
\gamma_{1}(\alpha)=\arcsin \left[\frac{F_{T}-F_{D}\left(\alpha, \gamma_{0}\right)}{F_{W}}\right] \tag{5.15}
\end{equation*}
$$

whether $\gamma_{1}<\gamma_{0}$ or $\gamma_{1}>\gamma_{0}$. According to the result, we increase or decrease the initial value $\gamma_{0}$ for the next round and calculate again $\ldots$ until $\gamma_{1}=\gamma_{0} \pm \Delta$ is fulfilled ( $\Delta=$ permitted deviation). We carry out this calculation for a desired thrust force in the entire angle of attack range; this is advantageously done by the PC in a mathematical programme.

Analogous to the speed polar of a glider model in gliding flight, Figure 5.8c, the so-called climb polar can be calculated in this way. This is done in Figure 5.14 a, b for the already mentioned model RS3 for two assumed thrust force values $\mathrm{F}_{\text {th }}$.


Fig. 5.14 a) Climbing polar of a model $v_{c r}(v)$ for two assumed thrust force values $F_{T}=20 \mathrm{~N}$ und $\mathrm{F}_{\mathrm{T}}=10 \mathrm{~N}$ und b ) associated angle of climb $\gamma(\mathrm{v})$; run variable $\alpha=$ angle of attack. The tangent to the climb polar describes the maximum possible angle of climb: $\gamma_{\max }=\arcsin \left(\mathrm{v}_{\mathrm{cr}} / \mathrm{v}\right)$

It can be seen that the maxima for the highest rate of climb $v_{\text {cr_max }^{m}}$ are at higher path velocities $v$ than those for the maximum angles of climb $\gamma_{\text {max }}$. For example, for $\mathrm{F}_{\mathrm{T}}=10 \mathrm{~N}$ the following results are obtained:
greatest rate of climb $v_{\text {cri }}$ max $=5.3 \mathrm{~m} / \mathrm{s}$ at $\mathrm{v}=17.3 \mathrm{~m} / \mathrm{s}$ and greatest angle of climb $\gamma_{10 \max }=23 \mathrm{deg}$ at $\mathrm{v}=$ $7.6 \mathrm{~m} / \mathrm{s}$; for $\mathrm{F}_{\mathrm{T}}=20 \mathrm{~N}$ : $\mathrm{v}_{\text {cr2 } 2 \text { max }}=14 \mathrm{~m} / \mathrm{s}$ at $\mathrm{v}=22 \mathrm{~m} / \mathrm{s} ; \gamma_{2 \text { degmax }}=56 \mathrm{deg}$ at $\mathrm{v}=5.9 \mathrm{~m} / \mathrm{s}$.
However, we also see that the minimum path speed of the model for $\mathrm{F}_{\mathrm{T}}=20 \mathrm{~N}$ with $\mathrm{v}=5 \mathrm{~m} / \mathrm{s}$ falls below the minimum speed allowed for gliding flight $\mathrm{v}_{\min }=7 \mathrm{~m} / \mathrm{s}$, compare figures 5.8 a ) to c ). This is due to the fact that with increasing angle of climb $\gamma$ the lift generated by the aerofoil is allowed to become smaller and smaller (and becomes smaller and smaller), because the thrust force $\mathrm{F}_{\mathrm{T}}$ takes over the forward motion of the model more and more.

## 5.2-4 Hovern

For a long time, hovering flight in the model sector was reserved for helicopters. Due to the high torques of brushless motors, this is now also possible for fixed-wing models. Here, the propeller must deliver a thrust force that corresponds to the weight force of the model: $\mathrm{F}_{\mathrm{T}} \geq \mathrm{m}^{*} \mathrm{~g}$, see Figure 5.35 in Chapter 5.3-3 Drive Components / Motor and Propeller.

### 5.3 Propulsion

## 5.3-1 Thrust, power, efficiency

For the engine-driven flight phases (horizontal, climb), in addition to the knowledge of the thrust force $\mathrm{F}_{\text {th }}$ generated by the propeller, the output power $\mathrm{P}_{\text {out }}$ emitted by it is often of interest. We obtain this via the relations
to

$$
\begin{gather*}
W=F \cdot s, \quad P=\frac{W}{t}, \quad P=\frac{F \cdot s}{t}, \quad P=F \cdot v \\
P_{\text {out }}=F_{T} \cdot v \tag{5.16}
\end{gather*}
$$

For clarity, the power curves for the results of the horizontal flight, Figure 5.15a, and the climb, Figure 5.15 b , are shown. This shows that in horizontal flight, an output power of Pout $\leq 10 \mathrm{~W}$ is sufficient for the slow flight range $\mathrm{v} \leq 10 \mathrm{~m} / \mathrm{s}$, while for the - only theoretically possible - extreme fast flight of $\mathrm{v} \approx$ $70 \mathrm{~m} / \mathrm{s}$, a power of Pout $\approx 4 \mathrm{~kW}$ would be required. In climb flight, the thrust $\mathrm{F}_{\mathrm{T}}=10 \mathrm{~N}$ for the maximum rate of climb at $\mathrm{v}=17.3 \mathrm{~m} / \mathrm{s}$ would have to be generated with a power $\mathrm{P}_{\text {out } 10}=170 \mathrm{~W}$ and the thrust $\mathrm{F}_{\mathrm{T}}=20 \mathrm{~N}$ for the maximum rate of climb at $\mathrm{v}=25 \mathrm{~m} / \mathrm{s}$ with $\mathrm{P}_{\text {out } 20}=440 \mathrm{~W}$, Fig. 5.15 b .


Fig 5.15 a) Necessary power output (blue) for horizontal flight and b) for the climb regimes according to Figure 5.14 (dotted)

In order to be able to infer the required input power backwards, one must include the efficiencies of the components involved in the drive, Figure 5.16. These multiply:

$$
P_{\text {out }}=P_{\text {in }} \cdot \eta_{\text {st }} \cdot \eta_{\text {mot }} \cdot \eta_{\text {getr }} \cdot \eta_{\text {prop }}
$$

so that for the necessary input power applies

$$
\begin{equation*}
P_{\text {in }}=\frac{P_{\text {out }}}{\eta_{\text {st }} \cdot \eta_{\text {mot }} \cdot \eta_{\text {getr }} \cdot \eta_{\text {prop }}} \tag{5.17}
\end{equation*}
$$



Fig 5.16 Chain of action of an electric drive with its efficiencies [Sche02].
Typical values for the individual components are in the range of
Controller: $\quad \eta_{\text {ctr }}=0,85 \ldots 0,95$
Motor: $\quad \eta_{\text {mot }}=0,7 \ldots \quad 0,9$
Gearbox: $\quad \eta_{\text {gear }}=0,85 \ldots 0,95$
Propeller: $\quad \eta_{\text {prop }}=0,4 \ldots 0,7$

## Example 5.2

a) the mass of the indoor training model pusher_tiefdecker (pu_td) is $\mathrm{m}=170 \mathrm{~g}$ when using a 360 mAh battery. It is supposed to reach a speed of $\mathrm{v} \geq 5 \mathrm{~m} / \mathrm{s}$ within a distance of $\mathrm{s}=5 \mathrm{~m}$.

1. What propulsive force Fs is required for this? We consider the starting process approximately as a uniformly accelerated movement. After time $t$, the relationship $s=\left(a^{*} \mathrm{t}^{2}\right) / 2$ applies to the distance travelled, from which it follows for the acceleration $a=2 * s / t^{2}$. The speed increases linearly over time $t, v=a^{*} t$, thus $a=v / t$. By equating, we obtain the time after which the desired speed is reached: $t=2 \mathrm{~s} / \mathrm{v}=2 * 5 \mathrm{~m} / 5 \mathrm{~m}^{*} \mathrm{~s}^{-1}=2 \mathrm{~s}$. From this, the value for the acceleration is determined as $\mathrm{a}=2 * 5 \mathrm{~m} /(2 \mathrm{~s})^{2}=2.5 \mathrm{~m} / \mathrm{s}^{2}$. With $\mathrm{Fs}=\mathrm{m} * \mathrm{a}$ we obtain the required propulsive force as $\mathrm{F}_{\mathrm{th}}=$ $170 \mathrm{~g} * 2.5 \mathrm{~m} / \mathrm{s}^{2}=0.43 \mathrm{~N}$.
In chapter 8, under Selected Topics, it is shown which values were obtained by measurement: $\mathrm{a}=2.3 \mathrm{~m} / \mathrm{s} 2, \mathrm{~F}_{\text {th }}=$ 0.39 N ; the model took off after a distance $\mathrm{s} \approx 7 \mathrm{~m}$ at a time of $\mathrm{t}=2.5 \mathrm{~s}$, the velocity was $\mathrm{v}=5.4 \mathrm{~m} / \mathrm{s}$. This case also shows how useful an estimation with the help of approximation relations can be. That one can also calculate more precisely is shown in section 8.8. There, air resistance and rolling friction are taken into account.
2. The final speed of the pu_td was measured at $v=11.5 \mathrm{~m} / \mathrm{s}$. It is assumed that the equilibrium state $\mathrm{F}_{\mathrm{th}}=\mathrm{F}_{\mathrm{d}}$ according to Figure 5.10 is reached, i.e. a uniform movement is present. What output power is required for this speed and what efficiency of the drive results if $\mathrm{P}_{\mathrm{el}}=2.8 \mathrm{~A} * 7.4 \mathrm{~V} \approx 21 \mathrm{~W}$ was measured as the power input? From Eq. (5.10) the lift coefficient is $c_{1}=m * 2 g /\left(v^{2 *} A f^{*} \rho\right)=0.125$; from the polar diagram in Fig. 8.3 the associated drag coefficient is $c_{d p}=0.04$. Together with the "harmful resistance coefficient" $c_{d s}=0.01$, the result is $c_{d g e s}=c_{p}+c_{d s}$ $=0.05$. From equ (5.9), the resistance force is $\mathrm{F}_{\mathrm{d}}=\mathrm{m}^{*} \mathrm{~g}^{*} \mathrm{c}_{\mathrm{d} \_ \text {ges }} / \mathrm{ca}=0.67 \mathrm{~N}$. Since $\mathrm{F}_{\mathrm{d}}=\mathrm{F}_{\mathrm{th}}$, the power output of the propeller is $P_{\text {out }}=v * F_{T}=11.5 \mathrm{~m} / \mathrm{s}^{*} 0.67 \mathrm{~N}=7.7 \mathrm{~W}$. The efficiency of the drive is therefore $\eta=P_{\text {out }} / P_{\text {el }}=$ $7.7 \mathrm{~W} / 20.7 \mathrm{~W}=0.37$.
b) What input power is required to drive the RS3 motor glider if it is to
3. reach a maximum horizontal speed of $\mathrm{v}=20 \mathrm{~m} / \mathrm{s}$, so that it can move forward even in a strong headwind?

According to fig 5.11a) $\mathrm{v}=20 \mathrm{~m} / \mathrm{s}$ is achieved with an angle of attack of $\alpha=-0.5 \mathrm{deg}$; according to figure 5.15 a ) the propeller must deliver a power $\mathrm{P}_{\text {out }}(-0,5 \mathrm{deg}) \approx 100 \mathrm{~W}$;
2. reach a rate of climb $\mathrm{v}_{\mathrm{cr}}=10 \mathrm{~m} / \mathrm{s}$ ? According to Fig. 5.15b) this rate of climb is reached at $\mathrm{v}=17 \mathrm{~m} / \mathrm{s}$, the propeller must deliver a power Pout $\approx 300 \mathrm{~W}$.
We assume the following values for the efficiencies: $\eta_{\text {ctr }}=0.9 ; \eta_{\text {mot }}=0.8 ; \eta_{\text {gear }}=1$ (no gearbox present); $\eta_{\text {prop }}=$ 0.4 . Thus, according to $\operatorname{Equ}(5.17)$, for the larger value of the output power of 300 W , the input power become $\mathrm{P}_{\mathrm{el}}=300 \mathrm{~W} /\left(0,9^{*} 0,8^{*} 1^{*} 0,4\right)=1042 \mathrm{~W}=1,042 \mathrm{~kW}$.
This power can be generated, for example, by 5 LiPo cells, corresponding to a voltage level of $\mathrm{U}_{\mathrm{Akku}}=5 * 3.7 \mathrm{~V}=$ 18.5 V and a motor that can handle a current of $\mathrm{I}_{\max }=1.042 \mathrm{~kW} / 18.5 \mathrm{~V}=56.3 \approx 60 \mathrm{~A}$ for a short time. This drive configuration was realised in model for photographyRS3_fotfly [EK07]. The evaluation of flights with different propellers resulted in an overall efficiency range $P_{\text {out }} / P_{\text {el }}$ of $0.40 \leq \eta \leq 0.44$.


The model RS3_fotfly [EK07] during a test flight on the Elbe meadows in Steutz 2007
c) The Airbus A320 requires the speed $\mathrm{v}_{\mathrm{lift}}=270 \mathrm{~km} / \mathrm{h}=75 \mathrm{~m} / \mathrm{s}$ for take-off.

1. what thrust $F_{T}$ must the two jet engines generate during take-off to reach this speed over a distance of $s=1500 \mathrm{~m}$ in $\mathrm{t}=35 \mathrm{~s}$ ? The mass is $\mathrm{m}=77000 \mathrm{~kg}$. From the acceleration $\mathrm{a}=2 * \mathrm{~s} / \mathrm{t}^{2}=2 * 1500 \mathrm{~m} /(35 \mathrm{~s})^{2}=2.45 \mathrm{~m} / \mathrm{s}^{2}$ we calculate $\mathrm{F}_{\mathrm{T}}=\mathrm{m} * \mathrm{a}=187 \mathrm{kN}$; according to the data sheet [A320_b] the engines generate: 2 x 118 kN .
2. what thrust must the two jet engines generate during cruise?

In forums [A320_c] a glide ratio of $E=20$ is mentioned for the A320. This gives us for the drag force $F_{d}=$ $\mathrm{F}_{\mathrm{L}} / \mathrm{E}=\mathrm{m} * \mathrm{~g} / \mathrm{E}=\left(77000 \mathrm{~kg} * 9.81 * \mathrm{~m} / \mathrm{s}^{2}\right) / 20=37.8 \mathrm{kN}$. But because $\mathrm{F}_{\mathrm{D}}=\mathrm{F}_{\mathrm{T}}$, cf. figure 5.10 , the required total thrust is $\mathrm{F}_{\mathrm{T}} \approx 38 \mathrm{kN}$.
3. What is the efficiency of the jet engines if, according to the data sheet, the kerosene consumption (fuel_flow) at cruising altitude is $\mathrm{K}_{\text {fuel }}=2700$ litres $/ \mathrm{hr}$ ?
The cruising speed is $\mathrm{v}_{\text {cruise }}=900 \mathrm{~km} / \mathrm{h}=250 \mathrm{~m} / \mathrm{s}$, cf. example 5.1 d ). Thus the output power generated by both engines becomes $\mathrm{P}_{\text {out }}=\mathrm{F}_{\mathrm{T}}{ }^{*} \mathrm{~V}_{\text {cruise }}=38 \mathrm{kN} * 250 \mathrm{~m} / \mathrm{s}=9.5 \mathrm{MW}$.
The density of kerosene is $0.8 \mathrm{~kg} / \mathrm{litre}$, so the fuel consumption is $\mathrm{K}_{\text {fuel }}=0.8 * 2700=2160 \mathrm{~kg} / \mathrm{hr}$. The calorific value of kerosene is $\mathrm{C}_{\text {fuel }}=11.9 \mathrm{~kW} * \mathrm{hr} / \mathrm{kg}$. This gives the power of the burnt kerosene as $\mathrm{P}_{\text {ker }}=\mathrm{K}_{\text {fuel }} * \mathrm{C}_{\text {fuel }}=$ $2160 \mathrm{~kg} / \mathrm{hr}^{*} 11.9 \mathrm{~kW} * \mathrm{hr} / \mathrm{kg}=25.7 \mathrm{MW}$. The efficiency of both jets is thus $\eta=\mathrm{P}_{\text {out }} / \mathrm{P}_{\mathrm{ker}}=9.5 \mathrm{MW} / 25.7 \mathrm{MW}=0.37$.

## 5.3-2 Drive components / rechargeable battery (Accumulator)

Although drive tests with electric motors for model aircraft were already carried out in the 1970s by F. Militky and G. Bruß, it took until about 1990 before suitable batteries for direct current motors were available in the form of the nickel-cadmium chemical system. Also promoted by the development progress in mobile radio technology, these were replaced around the turn of the millennium by lithium-ion or lithium-polymer batteries with a higher energy density.


Fig 5.17 Mass and volume-related energy density of batteries under development [ NaWi 14 ]
Just for comparison: the energy density of petrol is $43 \mathrm{MJ} / \mathrm{kg}=12 \mathrm{kWh} / \mathrm{kg}$, that of the best LiPo batteries $240 \mathrm{~Wh} / \mathrm{kg}$; so there is still the "small" difference of a factor of 50 here! Current developments, however, allow us to look optimistically into the future: Lithium systems in combination with sulphur or oxygen have at least such a magnitude already in their sights, Figure 5.17.
The relationship between current yield and duration of current "delivery" of a rechargeable battery is given by its charge capacity C :

$$
C=I \cdot t
$$

In the case of a constant discharge current Io, this can flow during the discharge time

$$
\begin{equation*}
t_{D}=\frac{C}{I_{o}} \tag{5.18}
\end{equation*}
$$

The magnitude of the discharge current Io, which can be drawn, and the duration of the discharge $t_{D}$ called discharge time in technical jargon, are thus inversely related, Figures 5.18 a and b .


Fig 5.18 Theoretical discharge current available depending on the discharge duration to
a) shown on a linear scale, the result is a hyperbola
b) on a double logarithmic scale shown results in a straight line

This relationship is evaluated below for two specific batteries: one with a capacity of $\mathrm{C}=1000 \mathrm{mAh}$ and one with $\mathrm{C}=360 \mathrm{mAh}$, see Fig. 5.19. Here, the table values in the middle column indicate how long a desired current Io can flow during the discharge time $t_{D}$ according to calculation Eq. (5.18) and, on the other hand, which times resulted from the measurements, right column. The state "battery empty" was defined by the level $\mathrm{U}_{\min }=3.3 \mathrm{~V} /$ cell, to which the battery voltage drops during the discharge process, see the following discharge diagrams.

| Io/A | $t_{\mathrm{D}} /$ min <br> theoretisch | $\mathrm{t}_{\mathrm{D}} /$ min <br> gemessen |
| :---: | :--- | :--- |
| 1 | 60 | 60,1 |
| 2,5 | 24 | 23,4 |
| 5 | 12 | 11,85 |
| 10 | 6 | ----- |


| $\mathrm{Io} / \mathrm{A}$ | $\mathrm{t}_{\mathrm{D}} /$ min <br> theoretisch | $\mathrm{t}_{\mathrm{D}} /$ min <br> gemessen |
| :---: | :---: | :--- |
| 1 | 21,6 | 15 |
| 2 | 10,8 | 6 |
| 2,5 | 8,6 | 1,7 |
| 3 | 7,2 | 0,8 |





Fig 5.19 Two batteries with 2 cells each with regard to their discharge times $t_{D}$ in comparison:
a) brand new, $C=1000 \mathrm{mAh}, \mathrm{Ri}=30 \mathrm{~m} \Omega$;
b) "worn out" $\mathrm{C}=360 \mathrm{mAh}, \mathrm{Ri}=300 \mathrm{~m} \Omega$

We can see that a new battery can be calculated very well according to Eq (5.18). However, the capacity decreases with the number of charging cycles, so that only low operating times are achieved with an "aging battery", right column in table of Fig. 5.19b. The 360 mAh battery was usually used by the author during indoor RC training with the model pusher_td.

Apart from the capacity C of a battery, its internal resistance


Fig. 5.20 Battery equivalent circuit Ri is a further "invisible" parameter, Figure 5.20.
This parameter can also increase many times (up to 10 times!) over the course of the charging cycles of a brand-new battery. In addition, however, a low ambient temperature also noticeably increases the internal resistance. In such cases, it is recommended to keep the battery warm until it is used in the model. An increased internal resistance means that only a reduced proportion of the internal voltage $U_{i}$ is available for connected load, even in the "battery full" state.
Example: $\mathrm{Ri}=0.3 \Omega$, $\mathrm{Io}=3 \mathrm{~A}$ results in a voltage drop $\mathrm{U}_{\mathrm{Ri}}=$ 0.9 V . Related to the nominal voltage $\mathrm{U}_{\text {ratings }}=3.7 \mathrm{~V}$, this is a voltage loss of about $25 \%$.

Information on the care and safe use of LiPo's is given at the end of this chapter. The labels on the batteries are explained below. Here the manufacturer provides information on the size of the charging and discharging current, which must be observed. In most cases, these are printed on the battery in plain text. In other cases, it can be taken from the somewhat misleading C-rate value, see the following examples in Fig 5.21.

$2 \mathrm{~S}=2$ cells $=>7,4 \mathrm{~V}$; capacity $\mathrm{C}=0,8 \mathrm{Ahr}$, this results in " 1 C " $=>0,8 \mathrm{~A}$


2-1000: 2 cells $=>7,4 \mathrm{~V} ; \mathrm{C}=1000 \mathrm{mAhr}$ capacity $\mathrm{C}=1 \mathrm{Ah}$, from that , $1 \mathrm{C} "=>1 \mathrm{~A}$ max charge current: $3 \mathrm{~A}=>3 \mathrm{C}$ max continous current $3 \mathrm{~A}=3 \mathrm{C}$ max short burst current $6 \mathrm{~A}=6 \mathrm{C}$


max continous current: $25 \mathrm{~A}=>25^{*} 0.8=20 \mathrm{~A}$; max short burst current: $40 \mathrm{C}=>40 * 0,8 \mathrm{~A}=32 \mathrm{~A}$


2S1P/1000: 2 cells in series, no parallel connection /1000mAhr
capacity $\mathrm{C}=1 \mathrm{Ah}$, from that , $1 \mathrm{C} *=>1 \mathrm{~A}$
max charge current: 4A
max continous current 15 A

Lipo $2 \mathrm{~S} 1 \mathrm{P} / 1000 \mathrm{mAh}$
$\uparrow$
2 cells in series
no cells connected in parallel
and a final designation example:


Fig 5.21 Designation examples for LiPo batteries and their meaning
The Li-Po batteries mentioned so far are preferably used in model aviation to supply power to drive motors and receivers. A second category, the NiMH batteries, are more commonly used in transmitters. Figure 5.22 shows the Graupner mx-16 as an example. When it is switched on, a constant current Io $=$ 200 mA flows, which means that with the specified capacity of 2000 mAhr , an operating time of $\mathrm{t}_{\mathrm{D}}=$ $2 \mathrm{Ah} / 0.2 \mathrm{~A}=10 \mathrm{hr}$ can be expected. Here, too, practice shows that this time is not exhausted. In fact, the manufacturer has already set the alarm signal "battery low" at 4.6 V - corresponding to $1.15 \mathrm{~V} /$ cell - which results in an operating time of around $\mathrm{t}_{\mathrm{D}}=7 \mathrm{hr}$. The measured discharge characteristic curve for this is shown in Figure 5.24.

Modern chargers offer special charging methods for both types, which must be used; otherwise there is a risk of damage or even destruction of the battery. The charging of a LiPo is done according to the socalled constant current/constant voltage method: Until the final charging voltage of 4.20 V is reached, the constant current set on the charger flows. When the voltage value is reached, the charger successively reduces the current towards zero.

It switches off and signals "battery full" when the charging current has dropped to about $10 \%$ of the set value, see Fig. 5.24. Incidentally, it follows from this charging method that batteries with increased Ri should be charged with a reduced charging current, as they switch off reduced by the voltage drop URi and are charged less fully at the nominal charging current.

$\overline{\text { Fig 5.22 Power supply of the Graupner HoTT mx16 transmitter with NiMH battery pack; 4,8V; 2000mAhr }}$
The charging process for NiMH batteries is different: when the final voltage is reached, a slight gas development occurs, which in turn leads to a voltage reduction due to an increase in pressure. This reduction is used for a sudden current cut-off; in the case of the Graupner Ultramat 16 S charger at the value $\Delta \mathrm{U}=-20 \mathrm{mV}$, see Figs. 5.23b and 5.24.



Bild 5.23 Modern chargers offer the charging procedures required for LiPo and NiMH a) The Ultramat 16 S charges/discharges not only $\quad$ b) In delta-peak method, the unit disconnect abrupt $\mathrm{NiCd}, \mathrm{NiMH}$ and LiPo, four other Li types. The the charging current of the NiMH battery when data can be read out on the PC.

In addition, follow the instructions for the safe and careful handling of batteries, [Gie15], [AS19]:

- Only charge LiPos with chargers with balancers designed for this purpose;
- First charges with a maximum of 1 C charge current, i.e. 1500 mAhr capacity $=>1.5 \mathrm{~A}$, charging with 0.5 C to 0.8 C is more gentle;
- Do not charge the LiPo battery higher than 4.2 V per cell, end the charge of the NiMH battery with the Delta-peak method;
- for charging/discharging, the optimum temperature range of the LiPo battery is 20 ... 300C,
$\rightarrow$ On a cool flying day, keep the battery at "comfortable" temperature until immediately before use;
$\rightarrow$ let the battery cool down to "hand warmth" before charging;
- If possible, do not use more than $80 \%$ of the specified capacity; i.e. set the cut-off voltage of the regulator to 3.3 V , where possible to 3.4 V ;
- Do not overload the battery, it must not become hot; from $\mathrm{T} \geq 60^{\circ} \mathrm{C}$ it would be permanently damaged;
- put the battery in a protective bag while charging and never leave it unattended;
- If the battery is to be stored for a longer period - which is already the case after 2 to 3 days - set the battery to the charge or discharge to the "storage voltage" 3.8 V per cell;
- Do not continue to use the battery after a crash;
- Batteries with a "bloated belly" have developed gas and are dangerous - store and dispose of them safely;
- Always dispose of batteries properly: in discharged condition, with insulated contacts, at recycling centres.
chemical
Nikel-Metall-Hydrid (NiMH)
system
Energy density
MJ/kg
Wh/kg typical voltage values at single cell Self-discharge

Temperature range suitable for Lifetime
charge,
Strom Ic( t )
80 to 61
60 to 100
$\mathrm{U}_{\text {min }}=1,0$ (empty: $<1.1 \mathrm{~V}$ )
$U_{\text {ratings }}=1,2 \mathrm{~V}$;
$\mathrm{U}_{\mathrm{max}}=$ not defined ${ }^{*}$ )
0,5 ... 1 \% per day
$50 \%$ per month
$5^{\circ} \mathrm{C} \leq \vartheta \leq 30^{\circ} \mathrm{C}$
Transmitter
500 charge/discharge-cycles
*) measured: $1,2 \mathrm{~V}$ bis $1,55 \mathrm{~V}$

|------Ic=const---------------------|lC=0
Charging current switched of when ...
charge,
Voltage $\mathrm{U}_{\mathrm{c}}(\mathrm{t})$

... voltage drops $\Delta \mathrm{U} \geq 20 \mathrm{mV} \uparrow$
discharge,
Voltage $U_{D}(\mathrm{t})$


Graupner mx16: 4NH-2000mAhr. The discharge current of $\mathrm{lo}=0.2 \mathrm{~A}$ results in a transmitter operating time of $\mathrm{t}_{\mathrm{D}}=7 \mathrm{hr}$

## Li-Polymer (Li-Po)

140 to 153
130 to 240
$\mathrm{U}_{\text {min }}=2,8 \mathrm{~V}$ (empty: $\leq 3,0 \mathrm{~V}$ )
$U_{\text {ratings }}=3,7 \mathrm{~V} ; \Delta \mathrm{U} \leq 50 \mathrm{mV}$
$\mathrm{U}_{\text {max }}=4,2 \mathrm{~V}$ (full: $>4,15 \mathrm{~V}$ )
$10^{\circ} \mathrm{C} \leq \vartheta \leq 60^{\circ} \mathrm{C}$
Drive and receiver
150 bis 200 charge/discharge-cycles


Charging current is reduced to $0 \mathrm{~A} \ldots$

.... when reached $\uparrow U=U_{\max }$


LiPo 2S450mAh; disch. currrent lo= 4A, $\mathrm{Ri}=200 \mathrm{~m} \Omega$, cut-off voltage $U=6 \mathrm{~V}$, corresponding to $3 \mathrm{~V} /$ Zelle

Fig 5.24 Battery types used in model flying and their characteristics

## 5.3-3 Drive components / motor and propeller

The electric motor is an electromechanical converter: it converts electrical energy (Wh) or electrical power (W) into mechanical energy ( Nm ) or mechanical power ( $\mathrm{Nm} / \mathrm{s}$ ).

The DC motor in the classical sense is shown in Figure 5.25. A magnetic field is built up by the field magnet (permanent magnet or electromagnet). An electromagnet - the armature - is rotatably mounted in this field. It receives its current from a power source via sliding contacts, usually designed as carbon brushes. The current flow in the coil of the armature builds up a magnetic field, which creates forces between the field magnet and the armature. Equal magnetic poles repel each other, unequal magnetic poles attract each other. These repulsive and attractive forces between the magnetic poles cause the armature to rotate.


Bild 5.25 Design principle of a classic DC motor [LH]
a) In this position the coil is supplied with current
b) The armature overcomes such a "gap" position due to mechanical inertia

The process is explained in detail using the motor with only one armature and only one stator: In the illustration Fig. 5.25a), the coil wound continuously over the armature body is soldered with the end near the south pole to the upper left slip ring half, with the end near the north pole to the lower right; the motor rotates to the left. If the armature has rotated so far that the north and south poles are opposite each other, Figure 5.25 b), the slider is between the slip rings and no current can flow through the coil, the movement would stop. Now, however, the mechanical inertia takes effect: the armature "has momentum" and rotates a little further, whereby the other polarity of the current source is connected to the coil ends. The current now flows in the opposite direction, the magnetic field of the armature reverses polarity, the attraction/repulsion force that arises again turns the armature further. This is a mechanical reversal of polarity, also called mechanical commutation. In practice, motors are designed with several field magnets and a collector divided into a corresponding number of segments. In this way, the motor runs "smoothly" and start-up problems are avoided. Note: a brushed motor has only two connections.

In the case of today's brushless motors, Figure 5.26, the control is contactless via a 3-phase voltage signal generated by a controller and shifted by 120 deg . The motor would not run with DC voltage. Compared to the classic DC motor, the brushless motor is characterised by a high torque.


Bild 5.26 Example of a "brushless" external rotor motor [Bü11]
a) The stator consists of 12 armature windings
b) 14 permanent magnets rotate around the windings
c) 3-phase current signal, shifted by 120 deg for each of the 3 coil groups.

Figure 5.27 shows the Hacker motor A10-13L used in the Motorglider Strolch. The power supply is three-core, as required by the rotating field. The end shield carries the mounting cross, the coppercoloured part including the shaft form the body of rotation. The propeller is rigidly connected to the shaft by a clamping device.


Fig 5.27 Brushless out runner motor A10-13L and technical data specified by manufacturer [Hack19]. Note: a brushless motor needs three connections.

For the mathematical consideration, it is expedient to start from the power balance [MB10]. On the input side is supplied the electrical power $P_{e l}=U \cdot I$ (we neglect the "intermediate function" of the controller, which generates the signal backlash for the rotating field from the DC voltage of the battery). At the shaft, the motor "responds" with power components, which are composed of the electrical dissipation power $P_{d i s}=I^{2} \cdot R$, which may cause the armature winding to become warm, and the mechanical power $P_{\text {mech }}=c \cdot I \cdot 2 \cdot \pi \cdot n$ :

$$
\begin{equation*}
U \cdot I=I^{2} \cdot R+c \cdot I \cdot 2 \cdot \pi \cdot n \tag{5.19}
\end{equation*}
$$

Here U, I: motor voltage in V and current in $\mathrm{A}, \mathrm{R}$ : ohmic part of the armature winding in $\Omega$, c : Machine constant in Vs, n: rate of revolutions (revs) in $1 / \mathrm{s}$.
The mechanical power, in turn, consists of the power that can be tapped at the motor shaft $P_{s h}=M \cdot \omega=M \cdot 2 \cdot \pi \cdot n$ and of a mechanical power loss due to friction. M describes the torque of the motor in Nm. The friction can be seen from the fact that even for no-load operation, where no load is picked up at the rotating shaft, a no-load current Io is necessary to compensate for the friction losses of the motor. This value for the motor according to Figure 5.27 is $I_{o}=0,34 \mathrm{~A}$ at $\mathrm{U}=7,4 \mathrm{~V}$.
Dividing equation (5.19) by the current I , the speed can be determined with the manufacturer's
specification $k v=\frac{1}{c \cdot 2 \cdot \pi}$ at a known current, e.g. a measured value, as follows

$$
\begin{equation*}
n=(U-I \cdot R) \cdot k v \tag{5.20}
\end{equation*}
$$

Figure 5.28 a) shows the curve of the speed versus the motor current $n(I)$. The curve begins with the idling speed $n(I o)=n_{0}=10450 / \mathrm{min}$. If we load the motor, e.g. with a propeller, the rate of revolutions (revs) will decrease; a certain revs will be reached at a certain current. If we increase the load so much that the shaft finally stops turning, e.g. because we hold it firmly, the current flowing through the windings reaches its maximum value, the stall current $\mathrm{I}_{\mathrm{st}}=\mathrm{U} / \mathrm{R}=11 \mathrm{~A}$. The useful power in this case is also $\mathrm{P}_{\text {sh }}=0$ as in the case of no-load current. This relationship is described by the equation for shaft power

$$
\begin{equation*}
P_{s h}=(U-I \cdot R) \cdot\left(I-I_{o}\right) \tag{5.21}
\end{equation*}
$$

whose graphical course $\mathrm{P}_{\text {sh }}(\mathrm{I})$ is shown in Figure 5.28 b$)$. Also shown is the efficiency curve, which results from the ratio of shaft power to electrical input power:

$$
\begin{equation*}
\eta=\frac{P_{s h}}{P_{e l}} \tag{5.22}
\end{equation*}
$$



Fig 5.28 a) Curve of revolutions $n(I)$ vs. current I of the A10-13L; $U=7,0 \mathrm{~V}$

b) Shaft power $P_{\text {sh }}(\mathrm{I})$ and efficiency $\eta(\mathrm{I})$ vs. current of the A10-13L; $U=7 \mathrm{~V}$

The maximum shaft power is here with $\mathrm{P}_{\text {sh_max }}=18,3 \mathrm{~W}$ at about $\mathrm{I}\left(\mathrm{P}_{\text {sh_max }}\right)=5,7 \mathrm{~A}$, the maximum of the efficiency with $\eta_{\max }=0,69$ bei $\mathrm{I}\left(\eta_{\max }\right)=1,9 \mathrm{~A}$.

At this point we need to familiarise ourselves with the propeller. Figure 5.29 shows the analogy of a screw thread to the propelle "thread". We see that with the parameters diameter D and pitch H the blade angle of the propeller is also defined:

$$
\begin{equation*}
\beta=a \tan \left(\frac{H}{D \cdot \pi}\right) \tag{5.23}
\end{equation*}
$$



Fig 5.29 a) A helix is created by winding a wedge onto a cylinder of the diametre D . [Sche07].
b) During one revolution the propeller covers the pitch H .

However, this angle remains constant over the radius only for special helicopter rotors. Usually it changes from a large value close to the radius stadion $r=0 * R$ to a very small one at the end of the leaf, i.e. at $r=$ $1^{*} \mathrm{R}$, Figure 5.29. Therefore, that angle $\beta$ is considered for the slope $H$ which is measurable at the radius station $r=0.7^{*} R$.

You can also imagine - in a very simplified way - that the propeller pushes a column of air of length H backwards per revolution. This makes it possible to estimate the speed at which the column of air leaves the propeller:

$$
\begin{equation*}
v_{p r} \approx n \cdot H \tag{5.24}
\end{equation*}
$$

The modelling industry now offers a wide range of propellers: from miniature propellers with a diameter of about $\mathrm{D}=3 \mathrm{~cm}$ up to $\mathrm{D}=100 \mathrm{~cm}$, the pitch vary within a similar large range. For the aforementioned engine, the literature lists as possible propeller sizes $7^{\prime \prime} \times 4^{\prime \prime}, 8^{\prime \prime} \times 4^{\prime \prime}, 9^{\prime \prime} \times 5^{\prime \prime}$ where: $1^{\prime \prime}$ (say: 1 inch) $=$ $2,54 \mathrm{~cm}$. These three variables were therefore examined by the author with regard to their effect on the engine during stationary operation.


Bild 5.30: CFK- propeller from Aeronaut, varying in diameter D and pitch H .
From left: DxH = 9"x5"; 8"x4"; 7"x4"

For this purpose, the parameters current consumption, speed and static thrust have been recorded and shown in the diagrams in Fig. 5.31. It can be seen that the values of revolution $n(I)$ fit well into a straight line ${ }^{5}$. In addition to the current values, those for the other parameters of interest can also be read from this, see Table 5.1.


Fig 5.31 a) $U=7 V$; Measured revolutions values
vs. current; Prop $7 \times 4 / 8 \times 4 / 9 \times 4$
b) $\mathrm{U}=10,5 \mathrm{~V}^{6}$; Measured revolutions values vs.
current; Prop $7 \times 4 / 8 \times 4 / 9 \times 4$

[^4]The assessment of these values shall be made on the basis of a practical application: it shall be estimated whether the motor is suitable for the motor glider Strolch, Figure 5.37 . Example 5.3 will show that a shaft power $\mathrm{P}_{\text {sh }}=15 \mathrm{~W}$ is sufficient for this.

In table 5.1 we see that for a 2 S battery $(7 \mathrm{~V})$ the 8 x 4 or $9 \times 5$ propeller would be a good choice: the required shaft power is slightly exceeded and the power loss remains within limits with max. 16 W to 22 W . If we were to use a 3 S battery $(10.5 \mathrm{~V})$, a 7 x 4 prop would already deliver twice the required shaft power of $\approx 30 \mathrm{~W}$, but also with a larger electrical power input $\mathrm{P}_{\mathrm{el}}=54 \mathrm{~W}$. However, this value of around $70 \%$ would still be well below the maximum permitted, see the information in Figure 5.27.

| Propeller | 7 x 4 | 8 x 4 | 9 x 5 | 7 x 4 | 8 x 4 | 9 x 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{U}=7 \mathrm{~V}$ |  |  | $\mathrm{U}=10,5 \mathrm{~V}$ |  |  |
| param |  |  |  |  |  |  |
| $\mathrm{n} / \mathrm{min}^{-1}$ | 6984 | 6086 | 5143 | 8579 | 7350 | 6103 |
| I/A | 3,9 | 4,8 | 5,8 | 5,4 | 6,8 | 8,2 |
| $\mathrm{P}_{\mathrm{el}} / \mathrm{W}$ | 27,3 | 33,6 | 40,6 | 54 | 68 | 82 |
| $\mathrm{P}_{\text {sh }} / \mathrm{W}$ | 16,24 | 17,8 | 18,32 | 31,7 | 34,8 | 35,5 |
| $\eta / \%$ | 59,5 | 53 | 45,12 | 58,7 | 43,1 | 45,1 |
| $\mathrm{P}_{\text {dis }} / \mathrm{W}$ | 11,1 | 15,8 | 22,3 | 22,3 | 33,2 | 46,6 |
| $\mathrm{F}_{\mathrm{T} / \mathrm{gm}}$ | 120 | 160 | 200 | 200 | 250 | 300 |
| $\mathrm{F}_{\mathrm{T}} / \mathrm{N}$ | 1,18 | 1,57 | 1,96 | 1,96 | 2,45 | 2,94 |
| $\beta /$ deg | 10,31 | 9,04 | 10,03 | 10,31 | 9,04 | 10,03 |
| $\mathrm{vpr} / \mathrm{m} \cdot \mathrm{s}^{-1}$ | 11,6 | 10,1 | 10,7 | 14,3 | 12,25 | 12,72 |

Table 5.1 Measured values for speed n, current I and static thrust FT, Bild 5.32: peakTech_1560, which were obtained when the A-13L was loaded with the above- Adjustable power supply unit mentioned propellers in stationary operation. Values for power and $0-32 \mathrm{~V} / 30 \mathrm{~A}$ as "battery replacement' efficiency were from the diagrams Figure 5.31 a) and b).

However, the effort for such a metrological investigation is not small: a power supply unit, which in the present case supplies adjustable voltages $7 \mathrm{~V} / 10.5 \mathrm{~V}$ and output currents up to 15 A , figure 5.32 , as well as a reliably working tachometer, are prerequisites.

But can you also find the right propeller by calculation? To do this, you need to know how much power a propeller can absorb. Aeronaut publishes a diagram from which the measured values $\mathrm{P}_{\mathrm{pr}}(\mathrm{n})$ for different sizes of their propeller families can be taken, see figure 5.33. One can see, for example, that the power $\mathrm{P}_{\mathrm{pr}}=50 \mathrm{~W}$ can be absorbed by a propeller 9 x 5 at a speed $\mathrm{n}=7000 / \mathrm{min}$; a propeller of size 10 x 7 , however, manages this at $\mathrm{n}=5900 / \mathrm{min}$. The larger the propeller diameter D , the more shaft power it can absorb. For the concrete example, however, we also see a sufficiently exact match with the shaft powers $\mathrm{P}_{\text {th }}$ highlighted in green in table 5.1: for the propeller sizes $8 x 4$ with $n \approx 6300$ and $9 x 5$ with $n \approx 5200$, values $\mathrm{Ppr} \approx 20 \mathrm{~W}$ can be estimated from the graph.

The logarithmic ordinate scaling of the graph now already indicates a non-linear dependence on the speed. Analytically, this becomes clearer: for the power consumption of a propeller at static thrust, the following relationship applies [Sch03]

$$
\begin{equation*}
P_{p r}(n)=c_{p} \cdot \rho \cdot n^{3} \cdot D^{5} \tag{5.25}
\end{equation*}
$$

Here, $c_{p}$ : power coefficient of the propeller, $\rho$ : air density, n : Speed and D: Diameter of the propeller. We see that the speed is included in the power consumption to the third power, the diameter even to the fifth power.


Fig 5.33: On the power consumption of propellers $\mathrm{P}_{\mathrm{pr}}(\mathrm{n})$ according to [Aer19]

For the Aeronaut propeller family there is the approximation formula in the same bibliography

$$
\begin{equation*}
c_{p} \approx 0,833 \cdot \frac{H}{D}-0,0116 \tag{5.26}
\end{equation*}
$$

so that with (5.26) in (5.25) the power consumption $\mathrm{P}_{\mathrm{pr}}(\mathrm{n})$ can be calculated.
The different propellers installed on our example engine will now each take up a speed that results from the balance of offered shaft power $\mathrm{P}_{\mathrm{sh}}$ and absorbed power $\mathrm{P}_{\mathrm{pr}}$. Because the speed of the motor and propeller is the same,
and because also applies

$$
\mathrm{n}_{\mathrm{w}}=\mathrm{n}_{\mathrm{pr}},
$$

$$
\mathrm{P}_{\mathrm{w}}=\mathrm{P}_{\mathrm{pr}}
$$

$$
(U-I \cdot R) \cdot\left(I-I_{o}\right)=c_{p} \cdot \rho \cdot n^{3} \cdot D^{5}
$$

one can also calculate the speed for which this equation applies. That is, the zeros of the 3rd degree function must be determined in order to find those values of the current $\mathrm{I}(\mathrm{n})$ for which the speed n is equal to:

$$
\begin{equation*}
(U-I \cdot R) \cdot\left(I-I_{o}\right)-c_{p} \cdot \rho \cdot[(U-I \cdot R) \cdot k v]^{3} \cdot D^{5}=0 \tag{5.27}
\end{equation*}
$$

The solution can only be achieved numerically, advantageously again with the help of a mathematical program. In the graphical representation, the current zeros result as intersections of the characteristic curves of output shaft power $\mathrm{P}_{\mathrm{sh}}$ and input propeller power $\mathrm{P}_{\mathrm{pr}}$, see Table 5.2 and Figure 5.34. With the help of these power values, values for the thrust force $\mathrm{F}_{\mathrm{T}}$ can now also be estimated:

$$
\begin{equation*}
F_{T}=0,67 \cdot \sqrt[3]{\frac{\rho}{2} \cdot \pi \cdot D^{2} \cdot P_{\mathrm{sh}}{ }^{2}} \tag{5.28}
\end{equation*}
$$

Compared to the measured values, the calculated ones meet with useful low tolerances. However, one must not forget that the essential basis for this was laid with the measured values $n(I)$, which resulted in correction values for kv and R , see footnote 5 .


Tabelle 5.2 Values calculated from $\mathrm{Eq}(5.25)$ and their deviation from the measured values listed in Table 5.1

Fig 5.34: The propeller curves $\mathrm{P}_{\mathrm{pr}}(\mathrm{I})$ intersect the power curve $\mathrm{P}_{\text {sh }}(\mathrm{I})$

It has now been pointed out several times that the thrust values are those for static thrust. These values are interesting for the operation of aerobatic models, which have the so-called torquing or hovering in their programme, or for the operation of drones, Figure 5.35. In these cases the thrust $\mathrm{F}_{\mathrm{T}} \geq(2 \ldots 3) * \mathrm{~F}_{\mathrm{w}}$ should be designed, [Bü10].


Fig 5.35 a) Here A. Scheffler "hangs" his aerobatic model on the propeller - for that the static thrust is required

b) The DJI Mavic Mini drone is designed as a quadrocopter see the [DJ20]

The situation is different with a fixed-wing model. In flight, the difference between the jet velocity $\mathrm{v}_{\mathrm{pr}}$ generated by the propeller and the airspeed $v$ is smaller, and thus the thrust force $F_{T}$ is also smaller than when stationary. In the literature, the degree of progress $J$ is used in this case, a factor that relates the ratio of the airspeed $v$ to the circumferential speed of the propeller $v_{u}$ [Sche07], [Hep18]:

$$
\begin{equation*}
J=\frac{v \cdot 60}{D \cdot n} \tag{5.29}
\end{equation*}
$$

The diameter D is determined from a widely scattering range of $0.4 \leq \mathrm{J} \leq 0.8$. Further dimensioning methods are presented in section 7.5 / Selection of the drive.

Here we will first show that it is also possible to calculate with measurement results obtained from static measurements, as long as the flow at the propeller does not break off, i.e. the propeller angle of attack remains $\beta \leq 12 \mathrm{deg}$, compare Figure 5.2b and Table 5.1.



Fig 5.36 Climbing polar curve of the 300 g model Strolch. With a thrust of $\mathrm{F}_{\mathrm{T}}=1 \mathrm{~N}$ it reaches
a) a $\underline{\max }$ rate of climb $\mathrm{v}_{\mathrm{cr}}=2,7 \mathrm{~m} / \mathrm{s}$ at $\mathrm{v}=11,6 \mathrm{~m} / \mathrm{s}$; the angle of climb is $\gamma_{\mathrm{cr}}=13^{0}$ and the wing-angel of attac is $\alpha=-2,8^{0}$.
b) a max angle of climb $\gamma_{\mathrm{cr}}=18^{\circ}$ at $\mathrm{v}=5,7 \mathrm{~m} / \mathrm{s}$; the rate of climb is $\mathrm{v}_{\mathrm{cr}}=1,7 \mathrm{~m} / \mathrm{s}$ and the wingangel of attac $\alpha=4^{0}$.

Measurements on two projects worked on in connection with GTA_Modellflug show that for motor gliders of the class discussed here, a halving of the static thrust $\mathrm{F}_{\mathrm{T}}$ - can well be assumed for the thrust value in flight, see also sections 8.1 and 8.2 in the chapter Selected Topics. With the thrust specification $\mathrm{F}_{\mathrm{T}}=0.5 * 1.96 \mathrm{~N}$, the angle of climb $\gamma_{\mathrm{cr}}$ and the rate of climb $\mathrm{v}_{\mathrm{cr}}$ are calculated for the Strolch model according to chapter 5.2-3 Climb. It can be seen that the maximum values are reached at very different airspeeds $v$ of the model, figure 5.36; these must be flown via elevator trim.
Climbing measurements finally confirmed the calculated prediction of the required rate of climb $\mathrm{v}_{\mathrm{cr}}$, see Figs. 5.37. A practical application of the topic is described in Example 5.3.



Fig 5.37 Climb performance of the Strolch with 2 S battery. Onboard measurement ALTIS 4+
[Alt20]
a) Propeller $8 \times 4$ givs $v_{\text {cr }}=2,1 \mathrm{~m} / \mathrm{s}$
b) Propeller $9 \times 5$ givs $v_{c r}=3,4 \mathrm{~m} / \mathrm{s}$

With the Aerobtec Flight Manager the stored data can be read out


Fig 5.37c) Motorglider Strolch with Graupner mx16 R/C gear in instructor-pupil function.

## 5.3-4 Drive components / speed controller

This electronic component generates the aforementioned signals for the rotating field. A microprocessor controls the output stages in such a way that current flows through 2 groups of the 3 armature windings. Voltage signals for the speed and position of the armature are obtained from the 3rd group, which lead to a phase shift of the current signals in case of deviations. From this point of view, it is justified to speak of a speed controller.
When selecting the controller, it is important to take into account the requirements of the motor. For example, the motor should require a maximum of $80 \%$ of the continuous current specified for the controller. If, on the other hand, the model aircraft is to be flown at "half throttle" for a longer period of time - in the so-called partial load range - it must also be taken into account that this produces the greatest power loss in the controller output stage, i.e. the greatest heat generation. In this case, you should choose a controller with a nominal current that corresponds to twice the maximum motor current.


2 bis 3 Lipo
Imax_continous $=12 \mathrm{~A}$
Imax_short burst $=16 \mathrm{~A}$
BEC 5V/1A
programmable via transmitter or XQCard
LiPo-Abschaltspannun 3,0/3,2V einstellbar
Masse 5g
Bild 5.38 Brushless-Regler XQ12 und seine Technischen Daten [Pi16]

In most cases, the controller also supplies the 5 V voltage for the receiver, referred to as the BEC output (Battery Eliminator Circuit). Here, too, there are power considerations to take into account, hardly because of the few mA for the receiver, but rather the servo currents.
If only two servos with a total current of $\leq 0.5 \mathrm{~A}$ are to be supplied, then in the case of a linear LDO (Low-Drop-Out) voltage regulator, even with a 3 S battery, the power loss of $\approx 3 \mathrm{~W}$ remains within a tolerable range. When utilising the maximum current of $1 \mathrm{~A}-$ as is possible with the controller mentioned here - a 3 S battery, however, already produces 6 W of power loss, which can cause the tiny IC to become hot. It is not without reason that for larger models (e.g. a glider with 6 servos) the power supply for the receiver is employed with a separate - usually NiMH - battery recommended. In the case of the XQ12 controller, however, a switching regulator is installed which, due to its principle, only generates minimal power loss.


Bild 5.39 Programming card [Pi16]

Other interesting features of a controller are:

- Selectable cut-off level for the drive battery, for example 3.4V/cell for Lipo
- Reversing the polarity of the motor running direction, important if the controller-motor wiring has already been completed.
- Adjustment of an engine brake to avoid "free spinning" of the propeller after switching off.
- Timing in degrees $\approx 2$ xnumber of poles of the motor; Example: Hacker: 12 -pole external rotor $\Rightarrow$ 24deg ; recommended timing: 20deg to 25 deg

These parameters can be set (programmed) for each controller via the transmitter, whereby a prescribed sequence of (mostly throttle) stick movements must be adhered to - the controller acknowledges with beeps of varying lengths. Of course, it is more convenient to use a programming card, such as the XQ card from Pichler in Fig. 5.39; the function is self-explanatory and you do not have to "bend your ears" to catch every single note.

## Example 5.3

A popular competition task of our club was borrowed from the UHU Cup, a model aircraft competition of DAeC youth competitions: The climb phase of a motor glider with a maximum climb time of $\mathrm{t}_{\mathrm{cr}}=60 \mathrm{~s}$ is followed by a glide phase, which is to be completed with a flight time of $\mathrm{t}_{\mathrm{sr}}=180 \mathrm{sec}$ along a landing line. The altitude to be reached is equal to the altitude from which the glide should follow:

$$
v_{c r} \cdot t_{c r}=v_{s r} \cdot t_{s r}
$$

This results in the rate of climb to

$$
v_{c r}=v_{s r} \cdot \frac{t_{s r}}{t_{s t}}
$$

Assuming a descent rate of $\mathrm{vsi}=0.5 \mathrm{~m} / \mathrm{s}$, the rate of climb must then be at least

$$
v_{\mathrm{st}}=0,5 \cdot \frac{\mathrm{~m}}{\mathrm{~s}} \cdot \frac{180 \cdot \mathrm{~s}}{60 \cdot \mathrm{~s}}=1,5 \cdot \frac{\mathrm{~m}}{\mathrm{~s}}
$$

According to Eq. (5.11), the necessary propeller output power results in

$$
P_{\text {out }}=0,3 \cdot \mathrm{~kg} \cdot 9,81 \cdot \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \cdot 2 \cdot \frac{\mathrm{~m}}{\mathrm{~s}} \approx 6 \cdot \mathrm{~W}
$$

With an assumed propeller efficiency of $\eta \mathrm{pr}=0.4$, a shaft power $\mathrm{P}_{\mathrm{sh}}=15 \mathrm{~W}$ is required. The values for shaft power measured in table 5.1 confirm this estimate.

## 5.3-5 Drive components / selection criteria for the model aircraft drive system

Rule _1: One should follow the suggestions of the engine manufacturers. As an example, Table 5.3 gives details for the Hacker A10 series, of which the A10-L13 is covered in detail on the previous pages.

| Motor | A10-S |  | A10-L |  |  | Motor |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Leistungsbereich | max. $55 \mathrm{~W}(15 \mathrm{sec}$. |  | max. $75 \mathrm{~W}(15 \mathrm{sec}$. |  |  | Powerrange |
| E-Segler | bis $300 \mathrm{~g} / \mathrm{up}$ to $10,5 \mathrm{oz}$ |  | bis $500 \mathrm{~g} / \mathrm{up}$ to $17,5 \mathrm{oz}$ |  |  | Electric-Sailplane |
| Sport- und Scale | bis $250 \mathrm{~g} / \mathrm{up}$ to $8,8 \mathrm{oz}$ |  | bis $400 \mathrm{~g} / \mathrm{up}$ to 14 oz |  |  | Sport and Scale |
| Kunstflug und 3D | bis $170 \mathrm{~g} / 5,9 \mathrm{oz}$ |  | bis $200 \mathrm{~g} / \mathrm{up}$ to 7 oz |  |  | Pattern and 3D |
| Windungszahl | 12 | 15 | 7 | 9 | 13 | Turns |
| Empf. Prop-Größe | 6x3 | $7 \times 4$ | 7x3,5 | $8 \times 4,3$ | $8 \mathrm{x} 4,3$ | Recommended Prop Range |
| Zellenzahl | 2 LiPo | 2 LiPo | 2 LiPo | 2 LiPo | 3 LiPo | No. of Cells |
| zulässiger Dauerstrom [A] | 4A | 4A | 5,5A | 5,5A | 7 A | Continuous Current [A] |
| kurzzeitiger Strom ( 15 Sec ) [A] | 6A | 6A | 8A | 8A | 9 A | Max. Burst Current (15s) [A] |
| Leerlaufstrom (Io) @ $8,4 \mathrm{~V}$ [A] | 1,0 | 0,63 | 1,1 | 0,72 | 0,39 | $\begin{gathered} \text { Idle Current (Io) @8,4 } \\ \text { V [A] } \\ \hline \end{gathered}$ |
| Innenwiderstand (Ri) [Ohm] | 0,185 | 0,289 | 0,11 | 0,18 | 0,28 | Resistance (Ri) [Ohm] |
| RPM/Volt (Kv) | 2900 | 2320 | 2200 | 1700 | 1200 | RPM/Volt (Kv) |
| Rückwandbefestigung komplett | $\checkmark$ |  |  |  |  | Backmount complete |
| Prop-Adapter | APC and GWS |  |  |  |  | Prop-Adaptor |
| Befestigungsschrauben | $\square$ |  |  |  |  | Screws |

Tabelle 5.3 Manufacturer's instructions for A10 motors
Rule _2: One must be clear about the type of model for which the drive is to be configured. For this purpose, findings from formulae (5.24), 5.28) and table 5.1 can also be used:
*an increased diameter D results in an increased thrust $\mathrm{F}_{\mathrm{T}}$;
*an increased pitch $H$ results in an increased propeller jet speed $\mathrm{v}_{\mathrm{pr}}$.
In a series of contributions by Müller [Mü14], observations on parameters of drive configurations of different model types are recorded, which are explained below. They are also given in short form in Tab 5.4.

Motor glider: Model climbs with motor assistance to the height where thermals are expected. After switching off the motor, the model becomes a pure glider and remains in the air for a longer or shorter time, depending on the thermal situation. The engine running time is low compared to the gliding time; the engine can cool down again during the gliding phase. Under these circumstances, it is permissible for the model to climb with the specified short-term (usually 15 sec ) maximum current.
Parkflyer: Light and small model ( $b \leq 1.20 \mathrm{~m}$ ), which is controlled at low altitude in circles and loops. The motor usually runs for the entire flight time (10...15)Min. Here the drive must be configured so that the power loss remains small enough so that the motor does not overheat, or it must be cooled accordingly. Motor temperatures of $80^{\circ} \mathrm{C}$ can damage the magnets. But the power dissipation/heat generation of the controller must also be taken into account here, because especially with this type of model the partial load operation described above is predominant - plumes of smoke have already been observed on the flying model.

Trainer: Model with a larger wingspan ( $b \approx 2.50 \mathrm{~m}$ ), with which either thermal flight or simple aerobatics are practised. In the latter case, attention must also be paid to possible heating.

3D aerobatics: the motor must be able to pull the model vertically upwards in certain manoeuvres $(300 \mathrm{~W} / \mathrm{kg})$ or to torqued; in the latter case it must be noted that the cooling by the airstream is missing. Since the downward manoeuvres are mostly flown at idle, the motor will not reach its power limit on average.

Glider tow: Model with powerful engine and high static thrust, which can take off after a short taxiing time.

Pylon racing: a model designed for this purpose must be able to fly fast, its drive must still generate sufficient thrust at high speeds. This is achieved with a large pitch propeller, e.g. $5 \times 5$; speed pilots calculate with airspeed $85 \%$ of jet speed; $\mathrm{v} \approx 0.85^{*} \mathrm{v}_{\mathrm{pr}}$.

Drone: For good-natured controllability, as required e.g. for camera flights, the ratio of thrust to weight force $F_{t h} / F_{w}=2: 1$ is recommended. For drones with aerobatic characteristics, the ratio should be $3: 1$ or slightly higher.

|  | power to model mass | static thrust to weight force | climb rate $\mathrm{V}_{\text {sr }}$ | prop sped $\mathrm{V}_{\mathrm{pr}}$ | propeller pitch to Diameter |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | W/kg | $\mathrm{F}_{\mathrm{T}} / \mathrm{F}_{\mathrm{w}}$ | $\mathrm{m} / \mathrm{sec}$ | $\mathrm{m} / \mathrm{s}$ | H/D |
| Parkflyer | 100 | $\geq 0,4$ | 0,5... | $\leq 15$ |  |
| Motor glider | $140 . .170$ | $\geq 0,4$ | 2... 3 | 10... 25 | 0,6...0,8 |
| Hotliner | 500... 1500 |  | 5... 10 | $\geq 20$ | 1; i.e. $16 \times 16$ |
| Trainer | $100 \ldots 150$ | $\geq 0,6$ | 3... 5 | 14... 25 |  |
| dynam.acrobatic | 250 | $\geq 1,2$ |  | 15... 30 | 0,6...0,8 |
| 3D-acrobatic | 350... 400 | $\geq 2$ | 5 | 14... 22 | 0,5; z.B.13x6,5 |
| Seglerschlepp | 3000 |  | $3 . .4$ |  |  |
| slow oldtimer | 200 |  |  |  | $\leq 0,3$; z.B. $13 \times 4$ |
| Pylon racing | 250... 600 |  | 2 | $\geq 40$ | 1; z.B. 5 x 5 |
| Drone |  | $\approx 2$ |  |  | small |

Tabelle 5.4 Statistically recorded parameters on drive configurations [Mü14]

| Material | Density <br> $\rho$ | tensile strength $\sigma_{\mathrm{ts}}$ | compressible strength $\sigma_{p s}$ | bending strength $\sigma_{b s}$ | E-modulus <br> E | $\begin{gathered} \text { breaking } \\ \text { length } \\ \sigma_{\mathrm{bl}} \end{gathered}$ | ultimate strain $\varepsilon_{45}$ | temperature coefficient $\mathrm{T}_{\mathrm{C}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{cm}^{3}$ | $\frac{\mathrm{N}}{\mathrm{~mm}^{2}}$ | $\frac{\mathrm{N}}{\mathrm{~mm}^{2}}$ | $\frac{\mathrm{N}}{\mathrm{~mm}^{2}}$ | $\frac{10^{3} \mathrm{~N}}{\mathrm{~mm}^{2}}$ | km | \% | ppm |
| Styropor | $0.015 \ldots 0.02$ |  |  |  |  |  |  |  |
| Balsa soft | $0.6 \ldots 0.15$ |  |  | $8 \ldots 12.5$ | 2 | 12 |  |  |
| middle <br> hard | $\begin{array}{lll} 0.15 \ldots 0.25 \\ 0.25 \ldots .0 .4 \end{array}$ |  |  | $\begin{aligned} & 12 \ldots 20 \\ & 20 \ldots 40 \end{aligned}$ | $8$ | $\begin{aligned} & 15 \\ & 17 \end{aligned}$ |  | pos. |
| Pine middle | $0.4 \ldots 0.6$ | $70 \ldots 100$ |  | 40... 60 | 10 | 15 | 0.7 | pos. |
| hard | $0.6 \ldots 0.8$ | $100 \ldots 150$ |  | 60.200 | 20 | 18 |  |  |
| Plywood |  |  |  | 70 | 8 |  | 1 | pos. |
| Polyester resin | 1.24 | 50...70 |  | 60 |  |  | $1 . .3$ | pos. |
| Epoxy resin |  | 70... 90 |  | 80 |  |  | 1... 4 | pos. |
| Glass fiber roving fabric | $1.7 \ldots 2.5$ | $\begin{gathered} 800 \ldots 1000 \\ 300 \end{gathered}$ |  | $\begin{aligned} & 300 \ldots 450 \\ & 150 \ldots 200 \end{aligned}$ | $\frac{30 \ldots 50}{20}$ | 50... 70 | $\begin{aligned} & 2.5 \\ & 3.5 \end{aligned}$ | pos. |
| Carbon fiber roving fabric | $1.4 \ldots 1.7$ | 900... 1200 |  | 459...650 | $\begin{gathered} 90 \ldots 130 \\ 40 \end{gathered}$ | $\begin{gathered} 60 \ldots . \ldots 0 \\ 40 \end{gathered}$ | $\begin{aligned} & 1 \\ & 1.3 \end{aligned}$ | neg. |
| Synthetic fiber <br> roving <br> fabric | 1.25...1.4 | 130000 | 40000 |  | 60... 70 | 75...65 | 2.5 |  |
| $\begin{aligned} & \hline \text { Aluminium } \\ & \mathrm{AlCuMg} 2 \\ & 2024-\mathrm{T} 4 \end{aligned}$ | 2.8 | 300 |  | 100 | 70 | 16 | 20 | pos. |
| $\begin{aligned} & \text { spring strip } \\ & \text { steel } 55 \text { SiMn } 7 \end{aligned}$ | 7.8 | 1320 |  | 650 | 200 | 25...30 |  | pos. |
| Table 6.1 Parameters of materials that are interesting for model aircraft construction marked in grey. values apply for the resin composite |  |  |  |  |  |  | $\text { conversions: } 10 \mathrm{MPa}=\frac{1 \mathrm{kN}}{\mathrm{~cm}^{2}}=\frac{10 \mathrm{~N}}{\mathrm{~mm}^{2}}$ |  |

### 6.1 Strength parameters

In addition to the materials traditionally used for model aircraft construction, such as balsa and pine wood as well as covering paper, today covering foil, products made of glass and carbon fibres (fabrics, rovings), polyester and epoxy resin as well as foams (PUR, expanded polystyrene/styrofoam) are common. Most of the materials were used in the construction of gliders and large aircraft. Some materials have also been "discovered" by model sports enthusiasts from other fields for their needs. The covering foil was even developed especially for model aircraft construction.

Section 6.2 deals with the individual load cases on the flight model. In order to use the materials sensibly against these loads, it is good to know their most important properties. They are listed in table 6.1. Here are a few remarks: The density $\rho$ gives information about the mass contained per unit volume; it is given in $\mathrm{gm} / \mathrm{cm}^{3}\left(1 \mathrm{gm} / \mathrm{cm}^{3}=1 \mathrm{~kg} / \mathrm{dm}^{3}=1000 \mathrm{~kg} / \mathrm{m}^{3}\right)$. The tensile strength indicates at which tensile stress the material loses its strength. The compressive strength indicates the compressive stress at which the material is compressed. The unit of measurement for both quantities is $\mathrm{N} / \mathrm{mm}^{2}$ (we apply the force of 1 N when we hold a mass of exactly 101.97 gm in our hand and do not drop it). As can be seen from the table, wood can be loaded about twice as much in tension as in compression.
If a workpiece clamped on one side, e.g. a wooden beam, is loaded with a weight, it bends. This bending causes a stress $\sigma$ in the beam, it is maximum in the edge fibres: one as tensile stress (above), the other as compressive stress (below), see Fig.8b. The bending stress $\sigma_{b}$ now gives an indication of the maximum load-bearing capacity $\sigma_{b m a x}$ at which the bent workpiece breaks or bends (yielding). In terms of value, it lies between tensile and compressive stress. The still permissible bending stress is lower by a safety factor, for example $\sigma_{b_{-} \text {perm }}=\frac{\sigma_{b_{\max }}}{1.5}$. The unit of measurement is again $\mathrm{N} / \mathrm{mm}^{2}$.

### 6.2 Load cases

In a first approximation the wing ${ }^{7}$ counteracts the weight force $F_{w}$ of the model with its lift $F_{L}$. In the case of stationary gliding flight, this weight force is identical with the gravitational effect $\mathrm{F}_{\mathrm{w}}=\mathrm{m}^{*} \mathrm{~g}$. Other conditions arise in manoeuvre flight (turning, interception). In the case of a glider model, there is also the case of a rope take-off, see Fig 6.4.

First, the expected manoeuvre loads are to be estimated. If a model flies a loop with the speed v , the lift force $\mathrm{F}_{\mathrm{L}}$ must compensate for the centrifugal force $\mathrm{F}_{\mathrm{C}}$ in addition to the weight force $\mathrm{F}_{\mathrm{w}}$, Figure 6.1:

$$
F_{L}=F_{W}+F_{C}
$$

Compared to stationary gliding flight, the lift force is therefore higher by a certain factor, this is called the load factor or load multiple lv:

$$
\begin{gathered}
F_{L}=l v \cdot F_{W} \\
l v \cdot F_{W}=F_{w}+F_{C}
\end{gathered}
$$

With $F_{C}=\frac{m \cdot v^{2}}{r}$ we obtain

$$
\begin{equation*}
l v=1+\frac{v^{2}}{r \cdot g} \tag{6.1}
\end{equation*}
$$

[^5]While the "entry" velocity v can be determined according to Eq. (5.3), we have to estimate the looping radius r . An example may give a first impression of the size: $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}, \mathrm{r}=5 \mathrm{~m}$. We get

$$
l v=1+\frac{\left(10 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{5 \mathrm{~m} \cdot 9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=3,04
$$

The wing must therefore withstand a weight force of the model FG that is greater by factor 3 without breaking.
However, we can also determine the load that arises when the model is intercepted with the maximum possible lift $\mathrm{F}_{\text {Amax }}$ from a steeply inclined flight path. In this case

$$
\begin{equation*}
l v=\frac{F_{L \max }}{F_{W}} \tag{6.2}
\end{equation*}
$$

and with the maximum lift corresponding to Gl. (5.1) $\mathrm{F}_{\mathrm{Lmax}}=(\rho / 2) * \mathrm{v}^{2} * S^{*} \mathrm{c}_{\mathrm{max}}$ erhalten wir

$$
\begin{equation*}
l v=\frac{\rho}{2 \cdot g} \cdot \frac{c_{l \max }}{\frac{m}{S}} \cdot v^{2} \tag{6.3}
\end{equation*}
$$

If we now use $\operatorname{Eq}(5.3)$ for the velocity $v$ that the model would assume at a selected clmin, we obtain

$$
\begin{equation*}
l v=\frac{c_{l \max }}{\sqrt{{C_{l \min }}^{2}+{C_{d \min }}^{2}}} \tag{6.4}
\end{equation*}
$$

The relationship (6.3) is evaluated in Figure 6.2 for different area loads mass $/ S_{w}$ and for a $c_{a m a x}=1.1$. It can be seen that, for example, with a surface load of $40 \mathrm{~g} / \mathrm{dm}_{2}$ a load multiple of $\mathrm{l}_{\mathrm{v}}=13$ occurs, if a model flying with speed $\mathrm{v}=28 \mathrm{~m} / \mathrm{s}$ in path inclination flight is intercepted with $\mathrm{c}_{\operatorname{lmax}}=1.1$. Or more clearly: for a model with a fuselage mass of $\mathrm{m}_{\mathrm{f}}=1.2 \mathrm{~kg}$, about $13^{*} 1.2 \mathrm{~kg}^{*} 9.81 \mathrm{~m} / \mathrm{s} 2=153 \mathrm{~N}$ "pull" on the wing-fuselage connections when you pull it around from a nosedive with $\mathrm{c}_{\operatorname{lmin}}=0.084$ by "fully pulling" the elevator.


Fig 6.1: When intercepting, the airfoil must in addition to the weight force Fw also absorb the centrifugal force Fc


Fig 6.2: Load multiple Iv, which arises during the interception on Clmax of a model with speed v .

However, this situation is present in the turn in F3B speed flight (four distances, i.e. the glide ratio is at best still $\varepsilon=1: 5$, i.e. $\mathrm{E} \approx 5$ ). If we also consider the F3B competition practice of accelerating the model in a dive immediately before releasing the winch, we can expect load multiples of $1 \mathrm{v} \approx 40$ according to figure 6.2!

But the wing is also loaded in the classic launching with cable (towline), see the following pictures.


Fig 6.3a) Towline high launch. Assumption: the towline is anchored and the wind drives the model upwards. Theoretically achievable is $h_{\max }=\Delta \mathrm{s}^{*} \sin (\beta)$

a

Fig 6.4a) Hand tow with loose pulley. If the person is running at speed v , the model is moved at 2*v. [La91]


Fig 6.3b) High launch of RES models with bungee cord. Comparison flying 2020, mfcR airfield.


Fig 6.4 b) Winch launch with fixed pulley. In contrast to man-launched gliding, the winch here is in the pilot's position.

From considerations of the acting forces and velocities, Figs. 6.5a, b [Ho81], [La91], the following result for the lift force $F_{L}$ to be applied by the wing, which must essentially be absorbed by the spar, and for the model velocity

$$
\begin{equation*}
F_{L}(\beta) \approx F_{W}+F_{S} \cdot \sin (\beta) \cdot \frac{\cos (\beta)}{\cos (\delta)} \tag{6.5}
\end{equation*}
$$



Fig 6.5a): On the effect of tensile force in the towline launch. $\mathrm{F}_{\text {tl_ }}$, $\mathrm{F}_{\text {tI_ }}$ : horizontal and vertical components of the tensile force $\mathrm{F}_{\mathrm{tl}}$, [Ho81].

$$
\begin{equation*}
v(\beta)=v_{S} \cdot \frac{\cos (\beta)}{\cos (\beta-\delta)} \tag{6.6}
\end{equation*}
$$



Fig 6.5b): Decomposition into velocity vectors [La91]. $\delta=$ const, $=65 d e g$ : Angle between launch hook and fuselage longitudinal axis

During the take off process the towline angle $\beta$ changes . Figures 6.6 a, $b$ show how the lift force $F_{L}$ and the model velocity v change depending on this. Here $\mathrm{F}_{\mathrm{t}}$ is the estimated or measured towline pull force. The lift force $\mathrm{F}_{\mathrm{L}}$ reaches its maximum value $\left(\mathrm{dF}_{\mathrm{L}} / \mathrm{d} \beta=0\right)$ for a towline angle $\beta \approx 45^{0}$. Incidentally, the author still remembers well the sudden exertion required of the person dragging after the first few - still fast-running - metres!


Fig 6.6 a): Course of the lift force FL over the towline angle $\beta$. At maximum, the wing must withstand $F_{L}=557 \mathrm{~N}$ if $F_{t \mid}=400 \mathrm{~N}$.


Fig 6.6 b ): Model speed $v$ over the towline angle $\beta$ when the towline is pulled with $v=2 \mathrm{~m} / \mathrm{s}, 4 \mathrm{~m} / \mathrm{s}$ and $8 \mathrm{~m} / \mathrm{s}$

For the example of the F3J model RS3 developed in the mfcR, Figure 6.7a, the data gives:
$\mathrm{F}_{\mathrm{W}}=2.4 \mathrm{~kg} * 9.81 \mathrm{~m} / \mathrm{s}^{2}=23 \mathrm{~N}$, towline pull force $\mathrm{F}_{\mathrm{S}}=400 \mathrm{~N}$ (see measured value in Figure 6.7 b ), $\delta=$ 65 deg , the maximum lift force of 557 N (corresponding to $\mathrm{lv} \approx 24$ ). The fact that the buoyancy force exceeds the towline pull force can already be seen qualitatively in Fig. 6.5a): the pointer length for the lift force $\mathrm{F}_{\mathrm{L}}$ is clearly greater than that for the towline pull force $\mathrm{F}_{\mathrm{tl}}$. This is due to the different angles at which both forces act on the centre of gravity.
However, common competition practice deviates greatly from the classic high take-off model. For example, many F3J pilots do not steer their model upwards in a straight course, but build an S-shaped course into the flight path. And with this "sideways turn" the perlon towline is stretched again, which ultimately translates into additional height gain. In the force curve, this manifests itself in a "peak halfway", Figure 6.7b. The purpose of the measurement was to determine the maximum towline tensile force.


Fig 6.7a): F3J model RS3, which was designed and built with the significant participation of young club members of the $\mathrm{mfcR} ; \mathrm{m}=2,4 \mathrm{~kg}, \mathrm{~S}_{\mathrm{w}}=68 \mathrm{dm}^{2}$ [JW04]


Fig 6.7b) Measured towline pull $F_{t 1}$, three launching operations according to Fig. 6.4a). The load cell is mounted between the ground anchor and the towline [JW04]

### 6.3 Forces in the wing

Now the consequences resulting from the estimated loading cases for the required wing strength are to be considered. Assuming a symmetrical and quasi-rectangular lift distribution, Fig 6.8a, the strength model of the clamped beam applies to each wing half, Fig 6.8b. This is bent under the influence of the

a)

b)
c)


Fig 6.8 The bending moment decreases towards the wing tip
lift forces.
Over the half span the bending moment $\mathrm{M}(\mathrm{x})$ iscalculated as

$$
\begin{equation*}
M(x)=\frac{1}{2} \cdot \frac{l v \cdot m_{F} \cdot g}{2} \cdot \frac{b}{2}\left(1-\frac{x}{\frac{b}{2}}\right)^{2} \tag{6.7}
\end{equation*}
$$

and its course shows graphic Figure 6.8c.
It is largest at the clamping point $\mathrm{M}(\mathrm{x}=0)$ :

$$
\begin{equation*}
M_{\max }=l v \cdot m_{F} \cdot g \cdot b \cdot \frac{1}{8} \tag{6.8}
\end{equation*}
$$

The bending strength of the wing is essentially determined by its spar. For its design, it follows from the course of the bending moment that its cross-section at the wing root must be chosen large enough to withstand the moment Mmax.. It can be tapered towards the wing tip. The curved beam (spar) is subjected to compression on the upper side and tension on the lower side. The neutral line runs through the middle. The highest tensile and compressive stresses are therefore present in the edge fibres. It follows that the spar caps should be placed far outwards and chosen to be flat in cross-section. This allows a lot of material to reach the outside, see Fig 6.9. It is advisable to first select the width B of the (upper) spar cap so that it fits as well as possible into the airfoil contour. Its thickness which results from $0.5^{*}(\mathrm{H}-\mathrm{h})$, is then calculated according to $\mathrm{M}_{\text {max }}$.


Fig 6.9 Spar arrangement in the wing of the motor glider Strolch, airfoil S7012. The calculated value for the thickness of the top boom is $d=0.86 \mathrm{~mm} \approx 1 \mathrm{~mm}$; selected $d=2 \mathrm{~mm}$, see example 6.1. The superstructure aids ("protrusions" downwards) will be sanded away later, Fig. 6.11a).

For this purpose, we assume that the bending stress $\sigma_{\mathrm{b} \_ \text {perm }}$ of the material used must not be exceeded by the maximum occurring bending moment $\mathrm{M}_{\max }$ :

$$
\begin{equation*}
\sigma_{b_{-} \text {perm }} \geq \frac{M_{\max }}{W_{\text {spar }}} \tag{6.9a}
\end{equation*}
$$

From this we obtain the minimum necessary value for the section modulus W of the intended spar type,

$$
\begin{equation*}
W_{\text {spar }} \geq \frac{M_{\max }}{\sigma_{b_{-} \text {perm }}} \tag{6.9b}
\end{equation*}
$$

which we now have to dimension according to the specifications in Figure 6.10.


Fig 6.10 Moment of resistance $\mathrm{W}_{\text {spar }}$ of spar types

The procedure can be explained with three examples

## Example 6.1

The mass of the Motorglider Strolch is $m=0.3 \mathrm{~kg}$; of this the fuselage mass is $\mathrm{m}_{\mathrm{f}}=0.226 \mathrm{~kg}$, the wingspan is $\mathrm{b}=$ 11.2 dm . The steep banked flight is performed with [ $\mathrm{c}_{1 \_\min }=0.1 ; \mathrm{c}_{\mathrm{d}_{-} \min }=0.02$ ], the interception with $\mathrm{c}_{1_{\_} \max }=1$. According to Eq. (6.4), $\mathrm{lv} \approx 10$, with Eq. (6.8) $\mathrm{M}_{\max } \approx 40 \mathrm{Nm}$.
Because the spar caps is to be made of pine chords, we select the value $\sigma_{b}$ perm $=40 \mathrm{~N} / \mathrm{mm}^{2}$ for the top chord loaded in compression and bending according to Table 6.1. According to Eq. 6.9 b, this gives the required section modulus to $\mathrm{W}_{\text {bar }} \geq 40 \mathrm{Ndm} / 40 \mathrm{Nmm}^{-2}=100 \mathrm{~mm}^{3}$, which must be applied by the spar construction. The box spar is selected with the characteristic $\mathrm{a}=\mathrm{B}$, which results in the necessary required chord spacing

$$
\begin{equation*}
h=\sqrt[3]{H^{3}-\frac{M_{\max }}{\sigma_{b_{-} \text {perm }}} \cdot \frac{6 \cdot H}{B}} \tag{6.9c}
\end{equation*}
$$

With the further numerical values $\mathrm{B}=7 \mathrm{~mm}, \mathrm{H}=14.5 \mathrm{~mm}$ (airfoil S7012), you get the inner belt distance $\mathrm{h}=$ 12 mm , which results in a spar cap thickness of $0.5^{*}(\mathrm{H}-\mathrm{h})=1.25 \mathrm{~mm}$. Since wooden slats with a thickness of $\approx$ 1 mm are difficult to handle, the dimensions $(7 \times 2) \mathrm{mm}^{2}$ are chosen for both belts.

The practical spar construction is now done in such a way that balsa webs are glued between the chords, which - decreasing in thickness - roughly follow the hyperbolic course of the bending stress Fig. 6.8c: starting with 7 mm thick balsa for the first panels (therefore $\mathrm{a}=\mathrm{B}$ !) the thickness decreases to 2 mm towards the end of the wing. Jumps in strength are thus avoided and the spar does not become unnecessarily heavy. The balsa webs are milled in such a way that their grain is perpendicular between the chords. In this way they secure the flat chords against buckling and also counteract the shear stress,
see figure 6.8 b . If the web grain ran in the direction of the span, there would be a danger of it splitting when the wing bends. The ribbed wing in the shell and in the (almost) finished model are shown in pictures 6.11.


Fig 6.11a) The superstructure aids on the ribs of the wing shell are sanded away. The reinforcement between the spar chords is also clearly visible.. b) Almost finished model motor glider Strolch

The consequences of an incorrect spar design can be seen in figure 6.12 b . An originally assembled kit model did not survive its first thermal afternoon. The following models in the GTA range were then


Fig 6.12 A wrongly designed double-T spar made of pine slats in a construction kit model a) Belt above $3 \times 3$, belt below $3 \times 8$ standing. The $\quad$ b) The model was controlled out of a thermal in weaker slat would have belonged at the bottom, the stronger belt - installed flat - at the top. moderately steep path flight. The wing broke during the interception.
constructed with intermediate spar reinforcements: Balsa intermediate webs in the entire middle section and additional planking on both sides in the root area with plywood 0.6 mm , Figure 6.13 .


Fig 6.13 Continuous webs, double-sided spar planking in the middle area, linearly decreasing: bending stress jumps are minimised in this way.

The poplar plywood for the fuselage was also replaced by 3 mm balsa. The take-off mass of the model was reduced by 200 gm to 500 gm without any loss of robustness..

For larger models, spar composite materials are often used. In these cases, care must be taken to ensure that their breaking stress is achieved at as equal an elongation as possible. From this point of view, wood and carbon fibres, would be the most suitable, Table 6.1 However, the temperature coefficient of wood is positive, that
of carbon fibre negative, so that additional stresses can arise. However, this effect is weakened in the bond with epoxy resin.

## Example 6.2

On larger (electric) glider models, wings are usually designed to be plugged together for transport reasons. The centre section is connected to "ears" of roughly comparable span, Figure 6.14. Carbon rods/tubes, which are available in various dimensions, are used as connectors [R\&G20]. In this specific case, the carbon shell wing of the RS3 is to be replaced by a lighter ribbed wing with the AG25 airfoil. The data are $\mathrm{b}=34 \mathrm{dm}$, fuselage mass $\mathrm{mR}=1.26 \mathrm{~kg}$, load multiple $\mathrm{lv}=10$. The spar calculation for the wing centre section with the maximum occurring bending moment $\mathrm{Mmax} \approx 525 \mathrm{Ndm}$, Figure 6.8 a , and the value given for carbon $\sigma_{\mathrm{b} \text { per }}=500 \mathrm{~N} / \mathrm{mm} 2$ results in CFK chords of dimension $10 \mathrm{~mm} \times 1 \mathrm{~mm}$.

The seperation should be at $\mathrm{x} 1=7.3 \mathrm{dm}$ (first red mark at the centre piece end, Figure 6.14). According to $\mathrm{Eq}(6.7)$ the bending moment at this point is $\mathrm{M}(\mathrm{x} 1)=172 \mathrm{Ndm}$, see Figure 6.16a. It is to be determined graphically and mathematically which $V$-angle (dihedral) is possible. In analogy to Eq. (6.9c) we obtain the minimum strength of a carbon bar to be

$$
\begin{equation*}
D=\sqrt[3]{\frac{M(x 1)}{\sigma_{b_{-} p e r m}} \cdot \frac{32}{\pi}} \tag{6.9d}
\end{equation*}
$$

This results in a diameter $\mathrm{D}=7 \mathrm{~mm}$, selected $\mathrm{D}=8 \mathrm{~mm}$, as 7 mm is not included in the delivery programme. The graph shows that only a V-angle $\phi \leq 3 \mathrm{deg}$ can be realised, Figure 6.16b.


Fig 6.14 Planned ribbed wing for the RS3_electric, assembled from 3 parts


Fig 6.15 Root airfoil AG25 of the wing centre section. The 1.5 mm balsa planking extends over the carbon bars. For the spar calculation the heights $H=15 \mathrm{~mm}$ and $\mathrm{h}=13 \mathrm{~mm}$ apply.


Fig 6.16 a) Bending stress curve in the wing centre section. At the bending point $x 1=$ 7.3 dm is the moment $\mathrm{M}(\mathrm{x} 1)=172 \mathrm{Ndm}$. The round bar must withstand this.

b) carbon rod (blue), $D=8 \mathrm{~mm}$, in carbon sleeve (orange) $8 / 10 \mathrm{~mm}$. At the bending point $\mathrm{h}=11.7 \mathrm{~mm}$. It can be seen that only a dihedral -angle $\phi \leq 3 \mathrm{deg}$ is feasible.

## Example 6.3

The pusher low-wing model (pusher_td) was used by many generations of pupils for learning RC-flying in teacherpupil operation in the hall. It is exposed to two main types of load: the impact on the wall of the hall with subsequent uncontrolled spin and the aerodynamic load of a steep turn to prevent it from hitting the wall. But it must also be possible to intercept the glider after an unexpected engine stop during climb.
For the unavoidable wall stops during training, the construction type "protected motor" has proven itself; the EPP nose absorbs the stop force very well. The landing gear proved to be susceptible to breakage; however, in the event of damage, training operations could always be continued with the help of manual take-offs.

The wing made of 3 mm thick Depron cannot withstand the aerodynamic loads without reinforcement. It is to be proven that only two carbon bar caps with a cross-section of $3 \mathrm{~mm} \times 0.5 \mathrm{~mm}$, $\left(\sigma_{\mathrm{b} \_ \text {perm }}=500 \mathrm{~N} / \mathrm{mm}^{2}\right)$ glued on the upper and lower side at the same distance from the leading edge produce sufficient strength, Figure 6.17.

## The data are:

Wing span $\mathrm{b}=8.7 \mathrm{dm}$, wing area $\mathrm{S}_{\mathrm{w}}=17 \mathrm{dm}^{2}$; its airfoil is "curved plate", similar to Gö417a. mass m_pusher $=$ 170 g , of which the fuselage weighs 114 g . If we insert the coefficients for slow and fast flight $\mathrm{c}_{1 \text { max }}=0.94$ and $\left[\mathrm{c}_{\mathrm{lmin}}\right.$ $\left.=0.13 ; \mathrm{c}_{\mathrm{d} \min }=0.05\right]$, see $\mathrm{Eq}(6.4)$, we get

$$
l v=\frac{0,94}{\sqrt{0,13^{2}+0,05^{2}}} \approx 7
$$

The bending moment is maximum in the centre of the wing, according to Eq. 6.8 it is calculated as $\mathrm{M}_{\max }=1.3 \mathrm{Nm}$. With the specified permissible bending stress, the moment of resistance $\mathrm{W}_{\mathrm{b} \_ \text {perm }}=\mathrm{Mmax} / \sigma_{\mathrm{b} \_ \text {perm }}=2.54 \mathrm{~mm}^{3}$. This moment must be absorbed by the two spar caps. The calculation model is the spar type box, Figure 6.10 , with the particularity $\mathrm{a}=\mathrm{B}$ and the values $\mathrm{B}=3 \mathrm{~mm}, \mathrm{H}=4 \mathrm{~mm}, \mathrm{~h}=3 \mathrm{~mm}$ :

$$
W_{\text {spar }}=\frac{B \cdot\left(H^{3}-h^{3}\right)}{6 \cdot H}=\frac{3 m m \cdot\left(4^{3}-3^{3}\right) \mathrm{mm}^{3}}{6 \cdot 4 \mathrm{~mm}}=4,6 \mathrm{~mm}^{3}
$$

also
and thus

$$
\begin{aligned}
4,6 \mathrm{~mm}^{3} & \geq 2,54 \mathrm{~mm}^{3} \\
\mathrm{~W}_{\text {spar }} & \geq \mathrm{W}_{\mathrm{b} \_ \text {perm }} \quad \text { fulfilled }
\end{aligned}
$$



Fig 6.17 The "curved plate" (Gö417a) of Depron, 3mm thick, was used in all beginner's training models the airfoil. Although very simple, the spar "construction" makes the wing overdimensionally stiff.

Development steps Indoor flight training model
The pusher_low-wing model, Figure 6.17, was the result of several development stages, which resulted from the training experiences in the hall. The result was the low-wing pusher_Tandem2, a robust and yet slow-flying model. A classification in the speed range is shown in Figure 6.19.


Front-drive high-wing aircraft: slow but sensitive to impact ......

pusher_high-wing: (Ogar bond), more robust against wall impact but no longer slow


Low-wing pusher_tandem1: Slow, but poor curve stability

pusher_low-wing: robust against impact, not yet slow enough


Low-wing pusher_tandem2: slow and good curve stability

Fig 6.18 Development result: slow-flying, on/upstroke robust, good controllability


Fig 6.19 Relationship between minimum achievable air speed and wing load $\mathrm{V}_{\mathrm{min}}\left(\mathrm{m} / \mathrm{S}_{\mathrm{f}}\right)$, [La16]. Even in the practised depron construction method, it should be possible to achieve $v=2,5 \mathrm{~m} / \mathrm{s}$ : $\mathrm{mit} \mathrm{S}_{\mathrm{w}}=25 \mathrm{dm}^{2} ; \mathrm{b}_{\mathrm{w}}=11 \mathrm{dm} ; \mathrm{A}_{\mathrm{w}}=5 ; \mathrm{m}=88 \mathrm{~g}$.

### 7.0 Design of a model airplane

In the simplest case, a model aircraft is designed "over one's thumb". According to personal taste - and the outsider has no idea how much of this can be put into the appearance of a model - wings, fuselage and tailplane are proportioned to each other and given "sleek" outlines.
The qualified practitioner looks around at competitions or informs himself in model sport magazines. In doing so, he will choose the model that is closest to his aesthetic sensibilities from among the models that have been placed. In both cases, there is a chance that he will produce a model that flies. It is almost not possible to construct a flying model so badly that it does not fly at all.
However, there is also the possibility of designing a model using rules that are laid down in a scheme.
These are to be applied to the design of a beginner's motor glider.

### 7.1 Intended purpose

For a beginner model, performance considerations play a subordinate role. In the foreground are constructional and aeronautical aspects:
*simple construction with consideration of only little existing technological knowledge; robust construction because of presumably hard landings;
*non-critical flight characteristics, i.e. largely self-stable flying, good-natured reaction to control errors.

### 7.2 Wing geometry (span/ aspect ratio/ area/ angle of attack)

The size of a model aircraft usually refers to its wingspan. The motor glider to be designed here should have a manageable size $b_{w} \geq 1 \mathrm{~m}$, chosen $b_{w}=11 \mathrm{dm}$. The wing width - i.e. the airfoil length $1 \mu$ - now depends on the chosen wing aspect ratio $A_{w}$; in the case of the rectangular wing it is $l \mu=b_{w} / S_{w}$, see Eq. (5.7) and Fig. 7.1b).


This is where the Reynolds number Re comes into play: In contrast to the simplified representation in Figure 5.2 for the profiles NACA 0012 and ClarkY with only one Re number each, the diagrams turn out differently for lower and higher Re numbers in aerodynamic reality, see Figures 7.2a) to c). We can clearly see in diagrams b) and c) that the curves are shift towards higher drag values at lower Re numbers.


Fig. 7.2) Beiwerte-Verläufe des Profils S7012 für $\mathrm{Re}=200000 ; 100000 ; 50000$
| a) $\mathrm{C}_{\mathrm{I}}=\mathrm{f}(\alpha)$; Proximity line:
|b) $C_{d}=f(\alpha)$
| c) $\mathrm{Cl}_{\mathrm{I}}=\mathrm{f}\left(\mathrm{c}_{\mathrm{d}}\right)$; variable: $\alpha$ $\mathrm{ca}(\alpha)=(\alpha-\alpha \mathrm{o}) \cdot \mathrm{q} ; \alpha \mathrm{o}=-1.75 \cdot \mathrm{deg} ; q=0.12 \cdot \operatorname{deg}^{-1}$ |

The Reynolds number is directly proportional to the product of airfoil length and speed. For the field of model flight it can be quickly determined with a "fist" formula:

$$
\begin{equation*}
R_{e} \approx v \cdot l_{\mu} \cdot 70 \tag{7.1}
\end{equation*}
$$

where the airspeed v is entered in $\mathrm{m} / \mathrm{s}$ and the chord length of the model $1 \mu$ in mm . First of all, it is important that the Re-number estimated according to $\mathrm{Eq}(7.1)$ is not smaller than the smallest Re-number given for a selected airfoil; it can hardly be assumed how much the airfoil drag deteriorates further towards smaller Re-numbers. On the other hand, this means that for a given wing span $b_{w}$ the aspect ratio $\mathrm{A}_{\mathrm{w}}$ must not be arbitrarily large and that $\mathrm{v}_{\text {min }}$ must be used for the speed calculated according to Eq. 5.3.

For estimation we need the expected model size in the form of the parameters $b_{w}, S_{w}, A_{w}, m$, whereby Figure 7.3 can be helpful for mass estimation. There, the masses of motor gliders built in Modellflugclub Rossendorf are recorded as points $\mathrm{m}\left(\mathrm{S}_{\mathrm{w}}\right)$ depending on the area $\mathrm{S}_{\mathrm{w}}$; a regression line calculated from these points "squints" between them. For the wing area $15 \mathrm{dm}^{2} \leq \mathrm{S}_{\mathrm{w}} \leq 20 \mathrm{dm}^{2}$ envisaged for the planed Model Strolch, a mass $0.25 \mathrm{~kg} \leq \mathrm{m} \leq 0.5 \mathrm{~kg}$ was thus estimated, resulting in a reassuring value $\mathrm{Re}_{\mathrm{min}}=56000$ with regard to the selected profile S 7012 : the polars show $\mathrm{Re}_{\min }=50000$.

$$
x\left(A_{f}\right)=0.19 \mathrm{~kg}+0.024 \frac{\mathrm{~kg}}{\mathrm{dm}} \cdot A_{f}
$$

$$
\rho=1,23 \mathrm{~kg} \quad S_{w}=16,1 \mathrm{dm}^{2}
$$



$$
\begin{gathered}
\mathrm{m}_{\text {low }}=250 \mathrm{gm} \quad \mathrm{~m}_{\text {high }}=500 \mathrm{gm} \\
\mathrm{c}_{1}=1 \quad \mathrm{~cd}=0,1 \quad 1 \mu=160 \mathrm{~mm} \\
v_{\text {min_low }}=\sqrt{\frac{m_{\text {low }} \cdot g}{S_{w} \cdot \frac{\rho}{2}} \cdot \frac{1}{\sqrt{{c_{l}}^{2}+{c_{d}}^{2}}}}
\end{gathered}
$$

$$
\mathrm{v}_{\text {min_ } \_ \text {low }} \approx 5 \mathrm{~m} / \mathrm{sec}
$$

$$
\operatorname{Re}_{\mathrm{low}}=\mathrm{v}_{\mathrm{min} \_ \text {low }} * 1 \mu * 70=56000
$$

Fig. 7.3) Mass over wing area of realised Motor gliders in the Rossendorf model flying club

If we wanted to design a performance model, an optimisation calculation would be made at this point with regard to the best wing aspect ratio. Optimisation criteria are the parameters best glide and least sink, $[\mathrm{Br} /$ Frie98]. In the present case of the beginner model, however, we should not reduce the Renumber any further and be satisfied with the aspect ratio $\mathrm{A}=\mathrm{b}^{2} / \mathrm{S}_{\mathrm{w}}=7.5$.
Nevertheless, it makes sense to determine the total polar curve of the model and to derive the mentioned parameters from it. To do this, we must formulate the approximate curve $\mathrm{c}_{\mathrm{dp}}(\mathrm{Re})$ of the airfoil S7012 for each lift coefficient in the range $0 \leq \mathrm{c}_{1} \leq 1.1$ and, using the predefined parameters of aspect ratio and flight mass, iteratively determine that speed which corresponds to the valid Re number within the polar field. The mathematical details of this will not be discussed here. As a result, the black curve (marked with circles) in Figure 7.4 describes the dynamic polar curve of the airfoil determined in this way, i.e. the course of the lift and drag coefficients through the polar field of the airfoil according to Figure 7.2c, and we obtain the model polar, solid red curve in Fig. 7.4), which has now taken into account the total resistance according to $\mathrm{Eq}(7.2)$

$$
\begin{equation*}
\mathrm{c}_{\mathrm{dtot}}\left(\mathrm{c}_{\mathrm{a}}\right)=\mathrm{c}_{\mathrm{dp}}\left(\mathrm{c}_{\mathrm{l}}, \mathrm{Re}\right)+\mathrm{c}_{\mathrm{di}}\left(\mathrm{c}_{\mathrm{c}}, \mathrm{~A}_{\mathrm{w}}\right)+\mathrm{c}_{\mathrm{dh}} \tag{7.2}
\end{equation*}
$$

However, the wing aspect ratio $\mathrm{A}_{\mathrm{w}}$ also influences the angle of attack required for a desired lift coefficient: the smaller the aspect ratio, the larger it must be selected in order to achieve the same $c_{1}$ value as for the wing with infinite aspect ratio according to Figure 7.2:

$$
\begin{equation*}
\alpha_{A}=\alpha+c_{l} \cdot \frac{57,3 \mathrm{deg}}{\pi \cdot A} \tag{7.3}
\end{equation*}
$$

These correction values for the angle of attack are visible in Figure 7.4 (pink curve, marked with squares): the angle values for the model polar are larger than those for the infinite aspect ratio.


Fig. 7.4: Polar field for the airfoil at $\mathrm{Re}=200000 ; 100000 ; 50000$ (without marking). With the specified model target data, the dynamic profile polar $-\boldsymbol{O}^{-\infty}$ and the model polar $\square \square \square$; calculated from this. Run variable: Angle of attack $\alpha$. The larger angle values at the model polar curve take into account the finite aspect ratio $\mathrm{A}_{\mathrm{w}}=7.5$.

When calculating the model polars, the parameters of best glide and lowest sink are naturally also included: for each $c_{1}$ and $c_{d}$ value, the path speed already had to be determined, so it is easy to also output the sink rate $\mathrm{v}_{\text {sr }}$ and the glide ratio $\varepsilon$ or $\mathrm{E}=1 / \varepsilon$ in each case. Figure 7.5 shows the velocity polars calculated in this way. If we compare these with the curves shown in Figure 5.8c, d for a larger and heavier model - and without a dependency $\mathrm{c}_{\mathrm{dp}}(\mathrm{Re})$ having been taken into account there - it can be seen that the values for the lowest sink rate and the best glide are also at different path velocities here.


Fig. 7.5a Speed polar $\mathrm{v}_{\mathrm{si}}(\mathrm{v})$ and

b) course of the best glide $E(v)=1 / \varepsilon(v)$ of the Strolch model. The minimum sink rate $v_{s r}$ is reached at the path velocitie $v=5.3 \mathrm{~m} / \mathrm{s}$ (according to $c_{l}=0.9$ ), the best glide $\mathrm{E}=1 / \varepsilon$ at $\mathrm{v}=7 \mathrm{~m} / \mathrm{s}\left(\mathrm{c}_{\mathrm{a}}=0,5\right)$.

This circumstance is interesting for the respective trim settings: when cranking out a thermal bubble, the model should fly as slowly as possible, with the minimum possible sink rate. If, on the other hand, the sky is scanned for thermal bubbles, this should be done as quickly as possible with little loss of altitude, i.e. in the regime of the best glide ratio or slightly faster.
The two trim positions could be found, for example, with the help of a time measurement when flying over a defined distance, whereby the 30 m long landing lines used in the mfcR competitions could be used as a basis. The time intervals $\Delta \mathrm{t}_{\mathrm{sr}}=5.7 \mathrm{~s} \approx 6 \mathrm{~s}$ and $\Delta \mathrm{t}_{\mathrm{E}}=4.3 \mathrm{~s} \approx 4 \mathrm{~s}$ should be measurable. However, stationary flight conditions must be ensured - for example, an flyover after interception from a higher altitude would provide falsified results.

### 7.3 Angle of setting EWD

Figure 5.2 has already illustrated the difference between a symmetrical and cambered profile. In addition to the higher maximum lift coefficient, the cambered profile also has the special feature that it already generates lift at an angle of attack $\alpha=0 \mathrm{deg}$ and only reaches the lift coefficient $c_{1}=0$ at a negative angle value, the so-called zero lift angle $\alpha_{0}$. Figure 7.6 d ) shows this relationship on a wing with an infinite span, here $\alpha_{0}=-1.75 \mathrm{deg}$.

a) $\alpha=3.5 \mathrm{deg} ;$ angle generates $\mathrm{c}_{\mathrm{I}}=0.6$

b) $\alpha=0 \mathrm{deg}$; angle generates $\mathrm{c}_{\mathrm{I}}=0.2$

c) $\alpha=\alpha_{0}=-1.75 \mathrm{deg} ;$ angle generates $\mathrm{c}_{\mathrm{I}}=0$

d) Proximity characteristic curve $\mathrm{c}_{\mathrm{I}}(\alpha)$ of S 7012 fOr Aspect Ratio $\mathrm{A}_{w}=\infty$

Fig. 7.6 Angle of attack examples for $\mathrm{c}_{\mathrm{l}}=\mathrm{f}(\alpha)$ of a cambered airfoil

We now consider the combination of wing and tailplane and initially assume that the airflow $\mathrm{v}_{\infty}$, as shown in Figure 7.7a, remains parallel to the longitudinal axis of the fuselage. It is easy to see that the angle difference between the wing and the horizontal stabilizer is equal to the angle of attack of the wing, i.e. $\mathrm{EWD}=\alpha_{\mathrm{w}}$. And because the elevator has a symmetrical airfoil, it will generate neither lift nor down force.

However, the circulation flow shown in Figure 2.6 also acts on the free flow in front of and behind the wing, so that the horizontal tail plane is actually subjected to a downward flow, the so-called downwash. If the horizontal tail plane is not to generate lift or down force, it must be set to the downwash angle $\alpha_{\mathrm{hs}}=$ $\chi_{0}$, Figure 7.7 b . In this case, the setting angle difference is EWD $=\alpha_{w^{-}} \chi_{0}$. The same effect can be achieved if the EWD is kept the same by setting the angle of attack of the tail plane back to zero, i.e. $\alpha_{\mathrm{hs}}=$ 0 deg , and reducing the angle of attack of the wing by $\chi_{0}$, Figure 7.7 c . In terms of construction, this has the advantage that the tail plane can be installed parallel to the longitudinal axis of the fuselage, in this case parallel to the underside of the fuselage, Figure 7.8. As a result, the fuselage will be inclined tailheavy in flight by the angle smaller than $\chi_{0}$, but this is of no interest here. In performance models, care is taken to ensure that the fuselage flies as aerodynamically favourable as possible; in commercial aircraft, the arrangement is chosen so that the fuselage is horizontal in cruising flight.

a) EWD $=\alpha_{f}-\alpha_{h}=\alpha_{f}-0 \mathrm{deg}=\alpha_{f} \quad$ ex. $\alpha_{w}=3.5 \mathrm{deg}$ egibt EWD $=3.5 \mathrm{deg}$

b) $E W D=\alpha_{f}-\chi_{0}$
ex. $\alpha_{w}=3.5 \mathrm{deg} ; \chi_{0}=2.2 \mathrm{deg}$ ergibt EWD $=1.3 \mathrm{deg}$

c) $E W D=\alpha_{f}-\alpha_{h}$
ex. $\alpha_{f}=1.3 \mathrm{deg} ; \alpha_{h}=0 \mathrm{deg}$ ergibt $E W D=1.3 \mathrm{deg}$
Fig. 7.7 To explain longitudinal difference of angle wing - horizontal stabilizer

The angle $\chi_{0}$ is calculated as follows [Jä40]:

$$
\begin{equation*}
\chi_{0}=\alpha_{0} \cdot(1-\xi)+\alpha_{A} \cdot \xi \quad \text { (7.4a) } \quad \text { where } \quad \xi=1-\frac{0,73}{A_{w}} \cdot\left[1+\sqrt{1+\left(\frac{b_{w}}{2 \cdot r h}\right)}\right] \tag{7.4b}
\end{equation*}
$$

Using the numerical values from Figure 7.1b and the angle values $\alpha_{0}=-1.75 \mathrm{deg}$ the following is obtained * for the lowest sink rate: $\alpha_{\mathrm{A}}=8 \mathrm{deg}$, see Fig 7.5a, $\chi_{0}=5.6 \mathrm{deg}$ und $\alpha_{\mathrm{w}}=\mathrm{EWD}=8 \mathrm{deg}-5.6 \mathrm{deg}=2.4 \mathrm{deg}$. * for the best glide: $\alpha_{\mathrm{A}}=3.5 \mathrm{deg}$, see Fig $7.5 \mathrm{~b}, \chi_{0}=2.2 \mathrm{deg}$ and $\alpha_{\mathrm{w}}=\mathrm{EWD}=3.5 \mathrm{deg}-2.2 \mathrm{deg}=1.3 \mathrm{deg}$. The tailplane is installed as intended with a value of $\alpha_{w}=0 \mathrm{deg}$ to the longitudinal axis of the fuselage; however, unlike a performance model, a wing angle of attack for the phase of least sinking, i.e. $\alpha_{\mathrm{w}}=$ EWD $=2.4 \mathrm{deg}$, is provided for the beginner model. The phase of best glide can be found via the trim setting of the tailplane if you have the appropriate flying skills.


Fig. 7.8 Installed in the Strolch model: angle of setting EWD $=\alpha_{w}=2.4^{0} ; \alpha_{\mathrm{el}}=0^{0}$

### 7.4 Aerodynamic center and centre of gravity position, longitudinal stability

We now complete the wing-tail plane configuration with the centre of gravity CG, Figure 7.9. A body in space always rotates around its centre of gravity. As shown in Figure 4.1, it is the point through which the three imaginary axes of an aeroplane pass.
Here, however, we are only considering the rotation around the transverse axis, also known as pitching. We can define a neutral point N.P. with the position $\mathrm{x}_{\mathrm{N}}$ on both the aerofoil and the horizontal stabilizer: it is located on the aerofoil at $x_{N w} \approx 0.25 \cdot l_{\mu v}$, and on the horizontal tail plane at due to the symmetrical airfoil at $x_{N h s}=0,25 \cdot l_{\mu h s}$. It is characterised by the fact that a change in angle of attack $\Delta \alpha$ results only in a change in force $\Delta \mathrm{F}_{\mathrm{L}}$ at this point; a torque $\mathrm{cm}_{0.25}$ also present around this point remains constant. Let us now consider the effect of an upward gust $\Delta \alpha$ on the combination of wing and horizontal stabiliser. As a result of an additional force $\Delta \mathrm{F}_{\mathrm{Lw}}$, a clockwise moment around the centre of gravity will occur on the wing, $M_{w}=\Delta F_{L w} \cdot\left(x_{S}-x_{N w}\right)$, the model wants to rear up; the additional force $\Delta \mathrm{F}_{\text {Lhs }}$ on the horizontal stabilizer generates the torque $\Delta F_{L h s} \cdot\left(r h+x_{N w}-x_{S}\right)$, however, it rotates anti-clockwise, and with a comparatively longer lever arm. Whereas for stationary flight, the two moments acting in opposite directions of rotation are equal:

$$
\Delta F_{L w} \cdot\left(x_{S}-x_{N w}\right)=\Delta F_{L e l} \cdot\left(r h+x_{N w}-x_{S}\right),
$$

it is important in the event of a gust that the aircraft deflected about the lateral axis returns to the normal flight attitude as a result of the opposing reaction of the horizontal stabilizer. This ability is called longitudinal stability.


Fig. 7.9 A rearing up of the wing (= right-turning moment) due to an upward gust is counteracted by the horizontal tailplane with its left-turning moment. The settling time is $\Delta \mathrm{t} \approx 100 \mathrm{~ms}$

As a result, the described force changes of the aerofoil and tail plane can be summarised to the effect of a force change $\Delta \mathrm{F}_{\mathrm{L}}$ at a aerodynamic center of whole model neutral point NP, Figure 7.10.


Fig. 7.10 An overall neutral point simplifies the statement on longitudinal stability
The model is longitudinally stable if the centre of gravity SP is in front of the neutral point NP, Figure 7.11a; the additional force $\Delta \mathrm{F}_{\mathrm{L}}$ applied after deflection at the neutral point returns the model to its normal position.


Fig. 7.11a) The center of gravity is in front of the neutral point - the model is longitudinally stable
If the centre of gravity SP is behind the neutral point NP , the additional force $\Delta \mathrm{F}_{\mathrm{A}}$ will have a destabilising effect after a gust deflection, it will reinforce the rotation that has started, Figure 7.10b


Fig. 7.11b) The centre of gravity is behind the neutral point - the model has no longitudinal stability
To obtain a stability measure as a numerical value, the position of the neutral point $\mathrm{x}_{\mathrm{N}}$ must be calculated:

$$
\begin{equation*}
\mathrm{x}_{\mathrm{N}}=\mathrm{x}_{\mathrm{Nw}}+\Delta \mathrm{x}_{\mathrm{i}} \quad(7.5 \mathrm{a}), \text { where } \quad \Delta x_{i}=B e u \cdot V_{H} \cdot l_{\mu w}(7.5 \mathrm{~b}) \quad \text { and } \quad V_{H}=\frac{r_{h} \cdot S_{h s}}{l_{\mu w} \cdot S_{w}} \tag{7.5c}
\end{equation*}
$$

The quantity $\mathrm{V}_{\mathrm{H}}$ is referred to in the literature as the empennage volume, presumably because the numerator results in a three-dimensional value in itself and it also indicates its influence on the centre of gravity position. A long moment $\operatorname{arm} \mathrm{r}_{\mathrm{h}}$ and a large horizontal stabilizer surface $\mathrm{S}_{\mathrm{hs}}$ result in a large $\mathrm{V}_{\mathrm{H}}$ in $\mathrm{Eq}(7.5 \mathrm{c})$ and thus a shift of the neutral point towards the tail unit.
According to Musil [Mu78], a range of $0.55 \leq \mathrm{V}_{\mathrm{H}} \leq 0.7$ should be observed for thermal and slope flying models; with the values according to Figure $7.1 b, V_{H}=0.7$ results. For $A_{w}=7.5$ and $A_{h s}=3.35$, the correction factor $\mathrm{Beu}=0.42$ can be estimated from the diagram in Figure 7.12. This gives the neutral point position according to $\mathrm{Eq}(7.5 \mathrm{a})$ for the moment-replacement wing, Figure 7.1b, at $\mathrm{x}_{\mathrm{N}}=0.82 \mathrm{dm}$.


Fig. 7.12 The correction factor $\mathrm{B}_{\mathrm{f}}[\mathrm{Beu} 58]$
How far in front of the neutral point should the centre of gravity be? Figure 7.11a already shows that a greater distance $\mathrm{x}_{\mathrm{N}}-\mathrm{X}_{\mathrm{cg}}$ between the neutral point $\mathrm{N}_{\mathrm{P}}$ and the centre of gravity $C G$ results in greater longitudinal stability, and a smaller distance results in less longitudinal stability.
Derived from this consideration, the stability measure or Stability Margin is defined in relation to the
wing chord length $1_{\mu \mathrm{w}}$ :

$$
\begin{equation*}
S M=\frac{x_{N}-x_{c g}}{l_{\mu w}} \tag{7.6a}
\end{equation*}
$$

For commercial aircraft, a stability factor $\sigma$ of at least 0.05 is required; for gliders, it is in the range of $0.05 \leq \mathrm{SM} \leq 0.15$. For RC glider models, $0.1 \leq \mathrm{SM} \leq 0.2$ is common for normal flight regimes. In the speed flight of F3B models, the centre of gravity is often moved back to the neutral point. In this case, it is already a matter of flying in an indifferent balance position, which requires the pilot's utmost concentration and can possibly only be mastered by him because it is only of short duration ( $\approx 20 \mathrm{sec}$ ). However, RC models of other classes, e.g. F5x motorised gliders, are also flown at higher speeds in gliding flight in view of possible stronger winds, i.e. with a lower $c_{1}$ than required for the best glide, i.e. with a lower EWD and therefore with the centre of gravity moved back. However, the lower stability can only be mastered by experienced RC pilots.

The following is selected for the beginner model Strolch $\sigma \approx 0.15$. From $\operatorname{Eq}(7.6 a)$ we get

$$
\begin{equation*}
x_{S}=x_{N}-S M \cdot l_{\mu w} \tag{7.6b}
\end{equation*}
$$

and thus $x_{S}=(82-0.15 \cdot 151) \mathrm{mm}=59 \mathrm{~mm}$

Of course, the calculated centre of gravity setting must be checked during a model's entry phase. After a certain "feel-good" speed has been found with manual starts using the trim, it should be brought into a


Fig. 7.13 Interception test to assess the SP_position
steep inclined flight path at right angles to the wind from an appropriate height, Fig. 7.13, c. After a few seconds, take your finger off the elevator stick and "leave the model to its own devices". Depending on the position of the centre of gravity, it can take different flight paths $a, b$ or $c$. If it then climbs steeply upwards, case_a, the centre of gravity is too far forward and the EWD is slightly too high. If it continues to fly at virtually the same steep angle, case_c, the centre of gravity is almost at the neutral point, i.e. too far back, and the EWD is slightly too small; the flight attitude is indifferent. In both cases, the centre of gravity should be altered by changing the trim weights/shifting the battery to such an extent that only a moderate bounce as in case_b occurs. We understand the EWD change here as a slight trim change of the elevator.

However, there is also a recommendation for the size of the vertical stabiliser $S_{s}$ in the interest of yaw stability, [Lab 8] :

$$
\begin{equation*}
V_{V}=\frac{r_{v} \cdot S_{s}}{l_{\mu f} \cdot S_{f}} \tag{7.5d}
\end{equation*}
$$

whereby, for the sake of simplicity, it is assumed that the distance between the centre of the vertical stabiliser and the centre of gravity $\mathrm{x}_{\mathrm{CG}}$ is approximately the same as the stabiliser moment arm $r_{s} \approx r_{\text {. Following the literature with the value range } 0.02 \leq V_{V} \leq 0.05 \text {, the size }}$ of the vertical stabiliser $\mathrm{S}_{\mathrm{S}}=1.56 \mathrm{dm}^{2}$ yields $\mathrm{V}_{\mathrm{V}}=0.04$ and also conforms to the regulations.

### 7.5 Selecting the drive

Chapter 5.3-3/Motor and propeller explains the basic relationships between motor and propeller and is supplemented with rules for component selection. The following examples show that the drive and model can also be assessed with different initial data.

We will initially remain with the Strolch model, for which a required shaft power $\mathrm{P}_{\text {sh }}=15 \mathrm{~W}$ was calculated in example 5.3. With a further estimated motor efficiency $\eta_{\text {mot }}=0.7$ (see Figure 5.16), we are looking for a motor with a minimum required electrical power $\mathrm{P}_{\mathrm{el}} \geq 21 \mathrm{~W}$. For the Hacker motor A10-13L, the data sheet shows a maximum electrical power $\mathrm{P}_{\text {elmax }}=75 \mathrm{~W}(15 \mathrm{sec})$; with the specifications $\mathrm{I}_{\max }=9 \mathrm{~A}$ $(15 \mathrm{~s})$ and $\mathrm{I}_{\text {cont }}=7 \mathrm{~A}$, a continuous power $\mathrm{P}_{\text {el_cont }}=75 \mathrm{~W}^{*}(7 / 9) \approx 60 \mathrm{~W}$ can be estimated. There is therefore no risk of overheating, even without special cooling measures, as the motor is only operated at around one third of the permitted continuous electrical power. In addition, use in a motorised glider always provides sufficient cooling time during the thermal search or sailing time. For the selection of the propeller we also use the measured values, Table 5.1: $\mathrm{P}_{\mathrm{sh}} \approx 20 \mathrm{~W}$, revlotion at this operating point $\mathrm{n} \approx 5000 / \mathrm{min}$ to $6000 / \mathrm{min}$. If we use these values in the graph in Figure 5.33, we find characteristic curves for propellers $8 \times 7$ to $9 \times 4$, which roughly confirms the $8 \times 4$ and $9 \times 5$ propellers used for the measurements - and also used for flight operation.
A further limit must be checked: it must be avoided to come close to the speed of sound with the propeller circumferential speed, because otherwise performance-reducing aerodynamic effects occur at the blade tips. According to [MPX21], this should be at a maximum of half the speed of sound, i.e. for the speed

$$
\begin{equation*}
n \leq \frac{0,5 \cdot 330 \cdot \frac{\mathrm{~m}}{\mathrm{~s}}}{\pi \cdot D} \tag{7.7}
\end{equation*}
$$

The maximum measured revolutions on the Strolch motor A13-L for the 7 x 4 propeller is only $\mathrm{n}=$ $8579 / \mathrm{min}$, even for a battery voltage of 10.5 V , see Table $5.1 .17720 / \mathrm{min}$ are permitted according to Eq (7.7); there is also a factor of 2 of safety here.

From the approximation $\mathrm{Eq}(5.24)$, the pitch H of the propeller can be checked:

$$
\begin{equation*}
H \approx \frac{v_{p r}}{n} \tag{7.8}
\end{equation*}
$$

With the velocity of the air column $\mathrm{v}_{\mathrm{pr}} \approx 10 \mathrm{~m} / \mathrm{s}$, see Table 5.1 and $\mathrm{n}=6086 / \mathrm{min}, \mathrm{H}=4.5$ is obtained. This means that the shortlisted props 8 x 4 and $9 \times 5$ are also correct in terms of pitch; the measurements of the climb rate, Figures 5.37, confirm this.


Fig. 7.14 Motorsegler EasyStar 3 von Multiplex, [HebH10/20] und https://www.multiplex-rc.de/

In another example, we want to find out what climb rate $\mathrm{v}_{\mathrm{cr}}$ the EasyStar 3 from Multiplex can achieve. The model was presented in FMT_H10/2020.
The data of interest are: Mass $\mathrm{m}=0.8 \mathrm{~kg}$; wingspan $\mathrm{b}_{\mathrm{w}}=1.37 \mathrm{~m}$, wing area $\mathrm{S}_{\mathrm{w}}=28 \mathrm{dm} 2 \rightarrow$ Aspect ratio $\mathrm{A}_{\mathrm{w}}=6.7$. With these data and the assumption [ca; $\mathrm{cw}]=[1 ; 0.09]$ see Figure 7.4, a minimum sink rate according to $\mathrm{Eq}(5.4)$ is calculated as $\mathrm{V}_{\mathrm{sr}}=0.6 \mathrm{~m} / \mathrm{s}$.

A more detailed analysis, such as the calculation of the sink or rise polars, is not possible,
because the airfoil of the wing used is not known. The Roxxy C28-30-09 motor used, see Table 7.1, is operated with a 3 S battery. Here too, the manufacturer does not provide more detailed information, such as resistance R or no-load current Io. Estimates are nevertheless possible:


Tabelle 7.1 Technical data for the Roxxy C28-30 [MPX21]

As the no-load current Io is not known, we estimate the idle speed to be $\mathrm{n}_{\mathrm{o}} \approx \mathrm{Uac} * \mathrm{kv}=$ $10.5 \mathrm{~V}^{*} 1280 / \mathrm{Vmin}=13440 / \mathrm{min}$ using $\mathrm{Eq}(5.20)$. The operating speed should then be $\mathrm{n}>0.65 * \mathrm{n}_{0}$, the usual range is $0.75^{*}$ no $\leq \mathrm{n} \leq 0.85^{*}$ no. If we calculate with the upper value of 0.85 , we obtain an operating revolution of $\mathrm{n} \approx 11420 / \mathrm{min}$. This makes it possible to estimate the shaft power:

$$
\begin{equation*}
P_{s h} \approx \frac{1}{k v} \cdot I \cdot n \tag{7.9}
\end{equation*}
$$

Initially using the continuous current value $\mathrm{I}_{\text {cont }}=18.6 \mathrm{~A}$, the result is Pwdauer $\approx 160 \mathrm{~W}$. From $\mathrm{Eq}(5.11)$, the rate of rise is calculated as follows

$$
v_{c r}=\frac{P_{o u t}}{m \cdot g}-v_{s i}
$$

where $P_{\text {out }}$ is equal to the power output of the propeller: $P_{\text {out }}=\eta_{\text {prop }} * P_{\text {sh }}$. With an assumed propeller efficiency $\eta_{\text {prop }}=0.5$, this results in a climb speed $\mathrm{v}_{\mathrm{cr}}=10 \mathrm{~m} / \mathrm{s}$, which promises an impressive climb.

Could the model also torque? Using $\mathrm{Eq}(5.28)$, the static thrust is calculated as $\mathrm{F}_{\text {th_cont }}=8.7 \mathrm{~N}$, corresponding to holding a mass of $m=0.88 \mathrm{~kg} \ldots$ that would be to little. However, if we briefly increase the motor current to $\mathrm{I}_{15 \mathrm{~s}}=25 \mathrm{~A}$, the thrust increases to $\mathrm{F}_{\text {th } 15 \mathrm{~s}}=12 \mathrm{~N}$, a value that can also be found in the manufacturer's specifications, Table 7.1. The drive would hold a mass $\mathrm{m}=1.2 \mathrm{~kg}$, with which the EasyStar3 could very probably torque, provided the pilot is able to do so.
To summarise, however, the many assumptions that were made in this case should be pointed out: they can certainly have the effect of tolerances towards the "good side", e.g. that even $\mathrm{v}_{\mathrm{cr}}=12 \mathrm{~m} / \mathrm{s}$ is achieved for the climb, but they can also generate disappointments, which means that torquing is not possible, for example.

Another example concerns a larger CFRP glider, the Stratus 4000, [BauH8/20], presented in FMT_H8/2020. Its data are: $\mathrm{b}_{\mathrm{w}}=4 \mathrm{~m} ; \mathrm{S}_{\mathrm{w}}=86.2 \mathrm{dm}^{2} ; \rightarrow \mathrm{A}_{\mathrm{w}}=18.7 ; \mathrm{m}=4.88 \mathrm{~kg} ; \mathrm{m} / \mathrm{S}_{\mathrm{w}}=56 \mathrm{gm} / \mathrm{dm}^{2}$. Despite its high wing loading, the sink rate is still $\mathrm{v}_{\mathrm{sr}} \leq 0.5 \mathrm{~m} / \mathrm{s}$; however, with $\mathrm{v} \geq 10 \mathrm{~m} / \mathrm{s}$ the model is also travelling quite fast in thermals. Tight beards will be difficult to crank out.
The high-speed flight data described by the test author, which were recorded by GPS logger during interception from a steep inclined flight path, are interesting: $v=60 \mathrm{~m} / \mathrm{s}$, acceleration in the interception arc $a=10.24 \mathrm{~m} / \mathrm{s}^{2}$. With $\operatorname{Eq}(6.1)$, Figure 6.1 and $a=v^{2} / r$, the load multiple

$$
l v=1+\frac{a}{g}
$$

results in $1_{v} \approx 2$, so this was not a highly loaded strength case. The speed mentioned also allows you to get a feeling for the lift and drag coefficient with which the model flies. Assuming a trajectory inclination flight with the glide ratio $\varepsilon \approx 1: 2$, i.e. $\mathrm{E}=2$, we obtain from the transformation of $\mathrm{Eq}(5.3)$

$$
c_{l}=\frac{m \cdot g \cdot 2}{S_{w} \cdot \rho} \cdot \frac{1}{v^{2} \sqrt{1+\frac{1}{E^{2}}}}
$$

the values $c_{1}=0.023$ and $c_{d}=c_{1} / E=0.012$. A comparison with the polar curve of the beginner's motorglider, Figure 7.4, shows that this operating point lies on the dynamic polar curve of the airfoil in the angle of attack range $-0.9 \mathrm{deg} \leq \alpha \leq 0.75 \mathrm{deg}$. From this it could be concluded that the conventional assumption for the "harmful resistance": $\mathrm{c}_{\mathrm{ws}}=0,01=$ const. is no longer appropriate. Instead, a dependency $\mathrm{cws}(\operatorname{Re})$ should also be introduced for this.


b) Analysing the Hacker A40-14L-V4; U = 21.5 V

Fig. 7.15a) Stratus 4000 by Valenta [BauH8/20]
The model is motorised with a 275 g Hacker motor A40-14L-V4-14-Pole kv 355, [Hack21], which is powered by a 6 S LiPo-Akku 3800 mAh . The detailed manufacturer specifications $\mathrm{P}_{\text {ell } 5 \mathrm{~s}}=1,1 \mathrm{~kW}$; $\mathrm{kv}=$ $355 / \mathrm{minV} ; \mathrm{R}=50 \mathrm{~m} \Omega$; $\mathrm{I}=0,85 \mathrm{~A} @ 8,4 \mathrm{~V}$ and $\mathrm{U}=21,15 \mathrm{~V}$ allow a performance analysis, Fig. 7.15b. This results in $\mathrm{P}_{\mathrm{w} 15 \mathrm{~s}}=980 \mathrm{~W}$ at $\mathrm{I}_{15 \mathrm{~s}}=54 \mathrm{~A}$ with $\eta_{\text {mot }}=0.85, \mathrm{n}=6550 / \mathrm{Min}$.
The values for the propeller size can be surmised from the diagram Fig. 5.33: 16x13, 17x11 und 18x11; the evaluation of $\mathrm{Eq}(7.7)$ yields $\mathrm{D} \leq 19^{\prime \prime} \approx 18^{\prime \prime}$. The calculation according to the diagram given in [Schu80], Figure 7.17, and the formulae derived from it
$c_{s}=v \cdot \sqrt[5]{\frac{\rho}{P_{w h} \cdot n^{2}}} ; \quad \lambda\left(c_{s}\right)=0,124 \cdot c_{s}^{2}+0,365 \cdot c_{s}+0,033 \cdot \frac{1}{1+c_{s}} ; \quad D\left(c_{s}\right)=\frac{v}{\lambda\left(c_{s}\right) \cdot n}$
also leads to the result $\mathrm{D}=18,4^{\prime \prime}$, if twice the speed of thermal gliding is assumed for the desired speed in powered flight, i.e. $\mathrm{v}=2 * 10 \mathrm{~m} / \mathrm{s}=20 \mathrm{~m} / \mathrm{s}$. The pitch of the propeller then follows the approximation [Brü21]

$$
\begin{equation*}
\mathrm{H}\left(\mathrm{c}_{\mathrm{s}}\right)=\mathrm{D}\left(\mathrm{c}_{\mathrm{s}}\right) *\left(0,253 * \mathrm{c}_{\mathrm{s}}^{2}+0,0833 * \mathrm{c}_{\mathrm{s}}+0,3226\right) \tag{7.11}
\end{equation*}
$$

and results in $\mathrm{H}=10$ '". The practically realised propeller has the well-matching size 17 x 10 .

In a final case, a thermal motorised glider is described that can typically be used in the F5J competition class. The task here is to soar for 10 minutes from the lowest possible starting altitude and to complete the flight with a target landing at the end. A logger that flies with the model measures and records the maximum altitude that the model has reached during the engine runtime and up to 10 seconds after the engine stops. This altitude then has a negative effect on the points score: up to an altitude of 200 m , half a point is deducted and from 200 m , three points are deducted for every metre of altitude. It is therefore desirable to switch off the motor at low altitude and still achieve the flight time of 10 minutes as far as possible. The models are therefore designed for minimum sink rate and airspeed - this is achieved primarily by a low wing loading $\mathrm{m} / \mathrm{S}_{\mathrm{w}}$, see equations (5.3) and (5.4). However, the motor and battery can also be selected in such a way that only the maximum sensible height $\mathrm{h}=200 \mathrm{~m}$ is reached within the permitted motor runtime $\mathrm{t}_{\text {mot }} \leq 30 \mathrm{~s}$, which is equivalent to a climb speed of $\mathrm{v}_{\mathrm{cr}} \geq 7 \mathrm{~m} / \mathrm{s}$.

Technical data of the model Django, [BauH2/20], published in FMT_H2/2020: $b_{w}=3 \mathrm{~m} ; \mathrm{S}_{\mathrm{w}}=52,3 \mathrm{dm}^{2}$; $\rightarrow \mathrm{A}_{\mathrm{w}}=17 ; \mathrm{m}=1 \mathrm{~kg} ; \mathrm{m} / \mathrm{S}_{\mathrm{w}}=20.15 \mathrm{~g} / \mathrm{dm}^{2}$. This is a remarkably low wing loading for such a large model, which is also reflected in the low sink rate $\mathrm{v}_{\mathrm{sr}} \leq 0.3 \mathrm{~m} / \mathrm{s}$ and above all in the low airspeed $\mathrm{v} \leq$ $6 \mathrm{~m} / \mathrm{s}$. The drive unit is also lightweight: Leopard LC250-30 6pol 5550kv with MicroEdition 5:1NL/T from Reisenauer [Reis21]. It has a mass of just 65 g and still delivers a shaft power of $\mathrm{P}_{\mathrm{sh}}=250 \mathrm{~W}$ at $\mathrm{I}=$ 50 A , Fig. 7.16b. Applying Eq(5.11), this results in a rate of climb $\mathrm{v}_{\mathrm{st}} \approx 12 \mathrm{~m} / \mathrm{s}$, if the efficiency of the propeller $11 \times 22$ is assumed to be only a weak value $\eta_{\mathrm{pr}}=0.5$.
Technical data of the drive unit: $\mathrm{kv}=1100 / \mathrm{V}^{*} \min$, $\mathrm{Imax}=50 \mathrm{~A}, \mathrm{P}_{\text {elmax }}=370 \mathrm{~W}$ at 2 S battery; The manufacturer also provides information for five selected propellers, static thrust in stationary operation see Table 7.2:

## Antriebsdaten im Stand:

$2 \mathrm{~s} \cdot 7,4 \mathrm{~V}-9 \times 21,5 \mathrm{GM}$ CFK M35, $6536 \mathrm{rpm}, 37$ Amp. 271 Watt, VPitch $214 \mathrm{~km} / \mathrm{h}$, Eta 75,3\%
$2 \mathrm{~s} \cdot 7,4 \mathrm{~V}$ - 11x22 GM CFK M35, $5947 \mathrm{rpm}, 49$ Amp. 365 Watt, VPitch $199 \mathrm{~km} / \mathrm{h}$, Eta 69,7\%
2s - 7,4V - 12x8 RF F5J schma M25, 7069 rpm, 25 Amp. 187 Watt, VPitch $86 \mathrm{~km} / \mathrm{h}$, Eta 79,1\%
2s - 7,4V - 14x 7 GM CFK M25, 6700 rpm, 33 Amp. 245 Watt, Schub 1569g, VPitch $71 \mathrm{~km} / \mathrm{h}$, Eta $76,7 \%$
2s - 7,4V-16x8 GM CFK M25, $6133 \mathrm{rpm}, 45$ Amp. 336 Watt, Schub 2554g, VPitch $75 \mathrm{~km} / \mathrm{h}$, Eta $71,5 \%$
HINWEIS: Schubwerte gerechnet
Tabelle 7.2 Parameters of Leopard LC250-30 in stationary operation. Thrust values calculated [Reis21]

These values can be used to calculate the missing parameters. Based on the motor equation $\operatorname{Eq}(5.19)$, the following can be written for two different measured load values:

$$
\begin{aligned}
& U=I_{1} \cdot R+c \cdot 2 \cdot \pi \cdot n_{1} \\
& U=I_{2} \cdot R+c \cdot 2 \cdot \pi \cdot n_{2}
\end{aligned}
$$

The solution to this system of equations is

$$
\begin{equation*}
R=\frac{U \cdot\left(n_{2}-n_{1}\right)}{I_{1} \cdot n_{2}-I_{2} \cdot n_{1}} \tag{7.12a}
\end{equation*}
$$

$$
\begin{equation*}
c=\frac{U \cdot\left(I_{1}-I_{2}\right)}{\left(I_{1} \cdot n_{2}-I_{2} \cdot n_{1}\right) \cdot 2 \cdot \pi} \tag{7.12b}
\end{equation*}
$$

For 14 x 7 and 11 x 22 this results in the values $\mathrm{R}=42 \mathrm{~m} \Omega$ und $\mathrm{kv}=1 / \mathrm{c} 2 \pi=1115 / \mathrm{min} * \mathrm{~V}$. The latter value is very close to the speed constants specified in the data sheet. These values can also be used to calculate the third parameter, idling current $\mathrm{Io}=1.94 \mathrm{~A}$. This makes it easy to analyse the performance of the motor. Fig. 7.16b)


Fig. 7.16a): Django von www.sansibear.de


Fig. 7.16b): Analysis of Leopard LC250-30 6pol 5550 kv , gear reduction ratio $5: 1$; $\mathrm{U}=7.4 \mathrm{~V}$

Applying of $\mathrm{Eq}(7.10)$, (7.11) we obtain $\mathrm{P}_{\mathrm{sh}}(49 \mathrm{~A})=250 \mathrm{~W} ; \mathrm{n}(49 \mathrm{~A})=5047 / \mathrm{min}$ and with a desired velocity $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$ the propeller $\mathrm{D}=15,3^{\prime \prime} \times 6,8^{\prime}$ '. The 14 " x 7 " propeller was used in the FMT article mentioned above.
The following diagram is taken from the literature [Schu80]. It is based on NACA Report No. 350 from 1929.


Bild 2.10 Rechenblatt Nr. 1 für Luftschrauben

Fig. 7.17 Propeller calculation diagram from H. Schulze [Schu80]. The curves Ho/D =f( $\mathrm{c}_{\mathrm{s}}$ ) and $\lambda^{\prime} \mathrm{L}\left(\mathrm{c}_{\mathrm{s}}\right)$ were approximated with polynomials for the formulae (7.10) und (7.11).

### 8.0 Selected themes

### 8.1 Measurements on the indoor training model low-wing_pusher

The indoor training model low-wing_pusher was developed especially for GTA flight training, fig. 6.18. The beginners learnt the basics of remote-controlled model flight. It has proven its worth over several years of student training: it flies well, its manoeuvrability supports last (milli-) second rescues and the robust design copes well with wall impacts or other beginner crashes. Speed measurements were carried out as part of the training programme in December 2013 with student support. The parameters are: $\mathrm{m}=170 \mathrm{gm}, \mathrm{S}_{\mathrm{w}}=16.4 \mathrm{dm}^{2}$, $\mathrm{A}=4,5$.
Slow flight: over a distance of 30 metres, students recorded the flight time when flying over waymarks ( $10 \mathrm{~m}, 15 \mathrm{~m}, \ldots 30 \mathrm{~m}$ ). The model was flown at its minimum speed, which should be as constant as possible, in order to aim for a uniform movement. The slope of the linear interpolation (regression line) results in $\mathrm{v}_{\mathrm{sl}}=4.2 \mathrm{~m} / \mathrm{s}$ (the abbreviation sl stands for slow).
In horizontal flight, the lift force is equal to the weight force $F_{L}=F_{W}$, see Figure 5.10. The effective lift coefficient can be calculated from $\mathrm{Eq}(5.10)$ as follows


Fig 8.1: Time-distance diagram at minimum possible speed

$$
\begin{equation*}
c_{l_{-} s l}=\frac{2 \cdot m \cdot g}{\left(v_{-} s l\right)^{2} \cdot S_{w} \cdot \rho} \tag{8.1}
\end{equation*}
$$

The result is $\mathrm{C}_{1}$ sl $=0,94$
If conclusions are to be drawn about the drive, the resistance value $\mathrm{F}_{\mathrm{d}}$ must be known because $\mathrm{F}_{\mathrm{th}}=\mathrm{F}_{\mathrm{d}}$. To do this, we first determine the resistance coefficient, see $\operatorname{Eq}(7.2)$

$$
\mathrm{C}_{\mathrm{dtot}}=\mathrm{C}_{\mathrm{dp}}+\mathrm{C}_{\mathrm{di}}+\mathrm{C}_{\mathrm{wh}}
$$

According to the measuring polars, fig 8.3, a $\mathrm{Cdp}_{\mathrm{d}}=$ 0.035 applies to the airfoil Gö417a $(\operatorname{Re}=42000)$ at $c_{l}=0.94$. With $A_{w}=4,5$ we obtain for the induced resistance coefficient according to $\mathrm{Gl}(5.8)$

$$
c_{d i}=\frac{c_{l}^{2}}{\pi \cdot \Lambda}=(0,94)^{2} / 3,14 * 4,6=0,065
$$

Using the harmful drag coefficient $\mathrm{c}_{\mathrm{dh}}=0,01$, we obtain the coefficient value of the total drag in slow flight

$$
\mathrm{c}_{\text {dtot_sl }}=0,035+0,063+0,01=0,11
$$

and for the drag force according to $\mathrm{Gl}(5.2)$

$$
\left.F_{D}=\frac{\rho}{2} \cdot v^{2} \cdot S \cdot C_{\text {dtot_sl }}=\left(0,5 * 1,23 \mathrm{kgm}^{-3}\right)^{*}\left[\left(4,2 \mathrm{~ms}^{-1}\right)^{2} * 0,164 \mathrm{~m}^{2} * 0,076\right)\right]=0,195 \mathrm{~N}
$$

According to Figure 5.10, in steady level flight $\mathrm{F}_{\mathrm{S}}=\mathrm{F}_{\mathrm{W}}$ and therefore $\mathrm{F}_{\mathrm{S}}=0,195 \mathrm{~N}$. This propulsive force $\mathrm{F}_{\text {th_sl }}=0,195 \mathrm{~N}$ must be generated by the drive. The propulsive power results in $\mathrm{P}_{\mathrm{sl}}=\mathrm{v}_{\mathrm{sl}} * \mathrm{~F}_{\mathrm{th} \_\mathrm{sl}}=4,2 \mathrm{~ms}^{-1} * 0,195 \mathrm{~N}=0,82 \mathrm{~W}$.

With the coefficients $\mathrm{c}_{\mathrm{d}_{\text {_sl }}} \approx 0,11 ; \mathrm{c}_{\mathrm{a}_{-} \mathrm{sl}}=0,94$ we obtain the operating point 1 in the polar diagram fig 8.3
Acceleration phase: as a uniformly accelerated movement is to be expected here, a distance of 30 metres was now subdivided more finely and the roll path up to take-off with subsequent low-level flight was tracked over time. The measured values from at least 5 repetitions were analysed, Figure 8-2a. In this case, the interpolation through the measurement points (curve fitting) was used to find the acceleration value that showed the best path over time according to the relationship

$$
\mathrm{s}=\frac{\mathrm{a}}{2} \cdot \mathrm{t}^{2}
$$

It resulted in $a=2.3 \mathrm{~m} / \mathrm{s}^{2}$. The velocity (represented by the straight line) reaches the value $\mathrm{v}_{\mathrm{fa}}=11.5 \mathrm{~m} / \mathrm{s}$ after the time $t=5.5$ s (the abbreviation fa stands for fast). Figure 8.2 b now shows a calculation ${ }^{8}$ of the velocity and displacement curve, taking into account the increasing resistance of the model. The velocity no longer increases in the further course but reaches a stationary value; it remains at $\mathrm{v}=11.5 \mathrm{~m} / \mathrm{s}$ : the equilibrium between thrust and drag force assumed in Fig. 5.10 has been reached. On the other hand, this shows the limits of an approximation; it is always important to make an estimate for its range of validity ... in this case it is only correct for the first few seconds. The calculation of the transient process is more complicated.


Fig 8.2 a) Distance measurements during the starting process. Red curve: velocity $\mathrm{v}(\mathrm{t})$; blue curve: distance(t)


Fig 8.2 b) Calculation of the complete start process In the shaded area, the measured values are values according to Fig. 8.2 a).

With the found acceleration value $\mathrm{a}=2.3 \mathrm{~m} / \mathrm{s}^{2}$, the thrust force emitted by the propeller results in $\mathrm{F}_{\mathrm{th} \_\mathrm{fa}}=$ mass $* \mathrm{a}=170 \mathrm{~g} * 2.3 \mathrm{~m} / \mathrm{s}^{2}=0.39 \mathrm{~N}$. The propulsive power is $\mathrm{P}_{\mathrm{fa}}=\mathrm{v}_{\mathrm{fa}} * \mathrm{~F}_{\text {th_fa }}=11.5 \mathrm{~ms}^{-1} * 0.39 \mathrm{~N}=4.5 \mathrm{~W}$. In relation to cruising flight, the demand for thrust is therefore higher by a factor of 2 and for power by a factor of 5. This may remind us of our last medium- or long-haul flight: after climbing at full power, the aircraft becomes pleasantly quieter when it reaches cruising altitude.

Here, too, we want to determine a polar point: Again from $\operatorname{Eq}(8.1)$ we calculate $c_{1}$ this time with the velocity value $\mathrm{v}_{\mathrm{fa}}=11,5 \mathrm{~m} / \mathrm{s}$. This results in the lift coefficient $\mathrm{c}_{1_{-} \mathrm{fa}} \approx 0.13$, the corresponding polar point is $\mathrm{c}_{\mathrm{dp} \_\mathrm{fa}}=0.04$. Te total value is $\mathrm{c}_{\mathrm{dfa}}=\mathrm{c}_{\mathrm{dp} \_\mathrm{fa}}+\mathrm{c}_{\mathrm{dh}}=0.05$, see operating point 2 in the polar diagram Figure 8.2 This point is also plausible, so our rather coarse measurement method provides satisfactorily accurate values.

## What we can learn from this

Although students prefer to practise aerobatics or "aerial combat", they are prepared to take measurements if there is a chance that they will get a turn as a pilot. In this case, the challenge is to bring the model from the unusually high speed into a "calm path" after take-off and the initially flat (= slightly depressed) flight. With regard to the evaluation, there can be an aha experience when the acceleration value $\mathrm{a} \approx 2 \mathrm{~m} / \mathrm{s}^{2}$ is estimated "free eye"... where else in practice is there the possibility of metrological contact with this physical quantity?

It is also interesting to compare the thrust $\mathrm{F}_{\text {th_fa }}$ emitted by the propeller during taxiing and flying with the value measured in static thrust: $\mathrm{F}_{\text {th_fa }} / \mathrm{F}_{\text {th_st }}=0.39 \mathrm{~N} / 0.8 \mathrm{~N} \approx 0.5$. We will encounter this ratio again in experiment 8.2/take-off trolley.

[^6]

Fig 8.3: Wind tunnel results for the Cambered plate Gö417a, $\operatorname{Re}=42000$ bis 420000 by [Sz42].
The points entered $1\left[c_{a \_s l}=0,94 ; c_{w \_s l}=0,11\right]$ und $2\left[c_{a_{\_} \text {fa }}=0,13 ; c_{w-f a}=0,05\right]$
result from measured values, which were determined with the participation of the pupils of the Gymnasium Dresden- Bühlau in slow and fast flight of the low-wing_pusher model.
8.2 Motor glider Luxx on take-off trolley


Fig 8.4: Motor glider Luxx on take-off trolley

A student from the 8th grade chose the subject of flying for his physics project. As a practical contribution, he planned to add a landing gear to his motorised glider. However, he was persuaded to build a take-off trolley instead of the undercarriage [La18].
What should the trolley look like? For use on a grass runway a robust body and sufficiently large wheels were required. Ball bearings were to be user to minimise friction. And the angle at which the longitudinal axis of the body should lie in the trolley had to be clarified.

How large must be the sum of the angles $\gamma+\alpha$ so that the model get airborne at a desired speed $v$ ?
It lifts off when


$$
F_{L} \geq F_{W}, \text { where } \quad F_{W}=\text { mass } \cdot g
$$

From the lift formula $\mathrm{Eq}(5.1)$ one calculates the lift coefficient

$$
c_{l} \geq \frac{2 \cdot F_{L}}{\rho \cdot v^{2} \cdot S_{w}}
$$


...and from the curve $c_{1}$ vs. angle of attack $\alpha$ the required angle sum, Figure 8.5 is calculated.
Using the data of the model Luxx: mass $=0.47 \mathrm{~kg}$ $\mathrm{F}_{\mathrm{L}}=0.47 \mathrm{~kg}^{*} 9.81 \mathrm{~m} / \mathrm{sec}^{2}=4.58 \mathrm{~N}, \mathrm{~S}=0.21 \mathrm{~m}^{2}$,
the angle sum of angels are given:
e.g. velocity $6 \mathrm{~m} / \mathrm{sec}, 8 \mathrm{~m} / \mathrm{sec}, 10 \mathrm{~m} / \mathrm{sec}$ :

$$
\begin{array}{rlll}
\mathrm{v} & =\left(\begin{array}{lll}
6 & 8 & 10
\end{array}\right) \mathrm{m} / \mathrm{sec} \\
\rightarrow \mathrm{c}_{1} & =1.0 & 0.55 & 0.36 \\
\rightarrow \gamma+\alpha & =\left(\begin{array}{lll}
8.2 & 3.6 & \\
1.4
\end{array}\right) \mathrm{deg}
\end{array}
$$

Fig 8.5: Take-off trolley and approximated airfoil
The practical implementation revealed summe $\gamma+\alpha=7^{0}$. There is a question: is an accelleration to $\mathrm{v}=6$ $\ldots 8 \mathrm{~m} / \mathrm{sec}$ possible in a gym for testing? And would there still be enough space for a landing manoeuvre?

With $\quad \Delta t=\frac{\Delta v}{a} \quad$ in $\quad \Delta s=\frac{a}{2} \cdot \Delta t^{2} \quad$ it yields $\quad s=\frac{v^{2}}{2 \cdot a}$
The numerical example: Desired speed $v=7 \mathrm{~m} / \mathrm{s}$ and assumed acceleration $\mathrm{a}=2 \mathrm{~m} / \mathrm{s}^{2}$ leads to $\Delta \mathrm{s}=12.3 \mathrm{~m} \ldots$ a value that gives hope.

## The positive test with video proof:



Fig 8.6: Test with video proof: 3 take-offs and 3 videos with position-variation of center of gravity
The video observation shows: the model lifts off at the same point always! Just as it gets airborne and started to fly, longitudinal stability is affected by responsible parameters (e.g. angle difference wing to horizontal stabilizer and position of c.g.).

But: how fast was the model at the moment of take-off? The analysis of the video shows the parameters of interest: lift-off time $t_{l_{0}}=2,8 \mathrm{sec}$ and lift-off distance $\mathrm{s}_{\mathrm{lo}}=9 \mathrm{~m}$. By solving the equations for the uniformly accelerated motion

$$
v_{l o}=a \cdot t_{l o} \quad s_{l o}=\frac{1}{2} \cdot a \cdot t_{l o}^{2}
$$

we obtain $\quad v_{l o}=\frac{2 \cdot s_{l o}}{t_{l o}}=6,43 \frac{\mathrm{~m}}{\sec } \quad$ and thus also $\mathrm{c}_{1}=0,86$ and $\gamma+\alpha=7^{0}$

The result is plausible, it confirms the initially estimated speed range of $6 \ldots 8 \mathrm{~m} / \mathrm{s}$. However, in view of the approximate airfoil data and a simple measurement method, the calculated angle $\gamma+\alpha=7 \mathrm{deg}$ matches the installed angle $\gamma+\alpha=7 \mathrm{deg}$ suspiciously well!
Incidentally, it is not clear in this context whether the induced angle of attack, i.e. the effect of aspect ratio- in this case $\mathrm{A}_{\mathrm{w}}=8,57$ - must be taken into account in this dynamic start-up process. If so, a measurement result of $\gamma+\alpha \approx 9 \mathrm{deg}$ would have been expected..
In addition, the effect of measurement tolerances must also be considered: if a tolerance of only $5 \%$ is assumed for time and distance measurement, $\mathrm{F}_{\mathrm{s}}=\mathrm{F}_{\mathrm{t}}= \pm 5 \%$, speed is distorted by $\pm 10 \%$ due to $\mathrm{F}_{\mathrm{v}}=\mathrm{F}_{\mathrm{s}}+$ $\mathrm{F}_{\mathrm{t}}$, resulting in the angles $5,5 \mathrm{deg}$ and $9,5 \mathrm{deg}$. For tolerance calculation, see section $8.5 /$ Tolerance considerations

Additional parameters can be calculated beyond the scope of the student's work:
*acceleration value $\mathrm{a}=2,29 \mathrm{~m} / \mathrm{s}^{2}$;
*Thrust force $\mathrm{F}_{\mathrm{T}}=($ mass_l $\mathrm{x}+$ mass_trolley $) * \mathrm{a}=(500 \mathrm{gm}+300 \mathrm{gm}) * 2,3 \mathrm{~m} / \mathrm{s}^{2}=1,76 \mathrm{~N}$;

* Power delivered by the propeller $\overline{\mathrm{P}}_{\mathrm{pr}}=\mathrm{v}_{\mathrm{lo}} * \mathrm{~F}_{\mathrm{th}}=6,43 \mathrm{~m} / \mathrm{s} * 1,76 \mathrm{~N}=11,3 \mathrm{~W}$;
*electrical power $\mathrm{P}_{\mathrm{el}}=7 \mathrm{~V} * 5 \mathrm{~A}=35 \mathrm{~W}$ and thus the total efficiency $\eta_{\mathrm{ges}}=\mathrm{P}_{\mathrm{pr}} / \mathrm{P}_{\mathrm{el}}=0,32$;
* the estimated distribution of efficiencies $\eta_{\text {contr }}=0,92 ; \eta_{\text {mot }}=0,9 ; \eta_{\mathrm{pr}}=0,4$;
* with measured static thrust $\mathrm{F}_{\mathrm{T}_{-} \mathrm{st}}=3,3 \mathrm{~N}$ the relation $\mathrm{F}_{\mathrm{T}} / \mathrm{F}_{\mathrm{T}_{-} \text {st }}=1,76 \mathrm{~N} / 3,3 \mathrm{~N}=0,53$.


## What we can learn from it

With simple measurements during take-off phase students can determine the physical variables velocity and accelleration at the take-off point. This corresponds to $9^{\text {th }}$ grade school physics.
The lift-off itself illustrates the term lifting force $F_{L}$
The model engine increased the trolley-speed linear up to the point where FA $>$ FG and the modell takes off. Using the measured values take-off-time and take-off-distance students can calculate the angle of attack $\alpha+\gamma$ during take off. They learn how to use the diagram Lift Coefficient vs. Angle of Attack.

Besides a link to theory, a trolley is additional fun during RC control training.

### 8.3 Take off with tricycle undercarriage

The following model was intended for transition training from beginner training - Figures 6.17 and 6.18 to indoor aerobatics. It was intended to introduce the students to aileron control. Nevertheless, it had to be impact-resistant, which is why the pusher configuration was also chosen here.


USA Lockheed P38 Lightning, first flight 1939. The aircraft was mainly used for long-range reconnaissance during the war.

Antoine de Saint-Exupéry, author of The Little Prince, did not return from a reconnaissance flight in such an aircraft in 1944.


Model: $b_{w}=1 \mathrm{~m} ; \mathrm{S}_{\mathrm{w}}=22 \mathrm{dm}^{2} ; \mathrm{A}_{\mathrm{w}}=4,6 ; \mathrm{m}=200 \mathrm{~g}$ tail plane $S_{h s}=1 \mathrm{dm}^{2} ; A_{\text {hs }}=2,7$

Fig 8.7 P38 aeroplane and training model with protected drive for aerobatic preparation
Like the aeroplane, the model also is designed with a tripod undercarriage, which gave rise to the question: What angle of attack $\alpha$ had to be installed in the model so that it would rotate around the main undercarriage by "pulling the elevator" and take off at $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$ ? Like the previous models, the wings and tail units were to be made of 3 mm thick Depron panels, but now the "flat panel" was to be used for the airfoil. The results of wind tunnel measurements are shown in Figure 8.9, [Sz42]. With the lever ratios shown in Figure 8.8, the model rotates over the main landing gear if
is fulfilled

$$
\begin{aligned}
& F_{h s}>m \cdot g \cdot \frac{x_{1}}{x_{2}} \\
& F_{h s}>0,8 \mathrm{~N}
\end{aligned}
$$



Fig 8.8 The take-off procedure consists of taxiing, rotating (increasing the angle of attack) and lift-off
If the horizontal tailplane were designed as a pendulum horizontal stabilizer, according to $\mathrm{Eq}(5.1)$ this value would be reached at a speed $\mathrm{v} \geq 16 \mathrm{~m} / \mathrm{s}$, assuming a $\mathrm{c}_{\_ \text {ns }}=0.5$. From Figure 8.9 we obtain a (negative) angle of attack $\beta \approx 5 \mathrm{deg}$ for that.


Fig 8.9 Wind tunnel results of the plane plate according to [Sz42]
For practical reasons, however, the tail plane was built with a flap. Musil provides a conversion diagram for this [Mu78], Figure 8.10; this applies in the linear range of lift behaviour. In this case, the ratio

$$
\frac{l_{\mu_{-} f}}{l_{\mu}}=\frac{39 \mathrm{~mm}}{105 \mathrm{~mm}}=0,37
$$

becomes the ordinate value $\frac{\partial \beta}{\partial \eta} \approx 0,7$ and we obtain the required flap deflection $\partial \eta=\frac{5 \mathrm{deg}}{0,7}=7 \mathrm{deg}$ instead of the pendulum elevator with the angular deflection $\beta \approx 5^{0}$.
However, if take-off is to take place at $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$, it is necessary to compensate the weight force $\mathrm{F}_{\mathrm{w}}$ with the aid of the counteracting wing lift force $F_{L}$ by means of the preset wing angle $\alpha$ to such an extent that the elevator deflection causes the model to rotate during taxiing.


Fig 8.10 Effect of the flap length $\mathrm{I}_{\mu_{-}} / /_{\mu}$ on the required flap angle [Mu78]

As with the horizontal tailplane, only a maximum lift coefficient of $\mathrm{c}_{\mathrm{lw}}=0.5$ can be generated at the wing. However, with $\mathrm{S}_{\mathrm{w}}=$ $22 \mathrm{dm}^{2}$, the surface area available for generating lift is around 20 times greater..
Even at $v=5 \mathrm{~m} / \mathrm{s}, \mathrm{F}_{\mathrm{L}} \approx 1.7 \mathrm{~N}$ is generated, even though the angle of attack required for $\mathrm{c}_{1}=0.5$ due to the only moderate aspect ratio $\mathrm{A}_{\mathrm{w}}=4.6$ according to $\mathrm{Eq}(7.3)$ requires an induced angle of attack of $\alpha_{\mathrm{Aw}}=7 \mathrm{deg}$.
With the realised angle of $\alpha=5 \mathrm{deg}$ in Figure 8.8, a satisfactory take-off procedure was achieved: after a few metres of taxiing, "pulling" the elevator led to rotation. The angle value was very easy to achieve with a slight extension of the nose wheel strut.

## What we can learn from it

A nose wheel landing gear extends the range of examples in the modelling area: in addition to the lift calculations for a wing, lever laws must now also be applied.

An elevator is a small aerofoil with a flap. A diagram is used to show how the parameters elevator flap and deflection angle can be approximately converted to the angle of attack of an undivided wing with the same lift force.

### 8.4 Calculation of speed polars in a Java programme

In this case, a model flight-based work entitled "Important aerodynamic and physical variables on an aerofoil" was produced for the subject Information Technology (IT). In the practical part, the student from a year 10 class used the Java-based development environment Processing to display lift and drag curves and the sink rate of a glider in the programme he created, Figure 8.11


Fig 8.11 Result output with the help of the VeloView programme created
The basic principles presented in the introductory section are taken from the literature, and some examples are also taken from the first version of this booklet accompanying the GT lessons.

## What we can learn from it

In this case, the application of GTA topics to school work relates to the IT subject. As done here, visualisation programs can initially be programmed to display characteristic curves; up to four parameters can be entered for this purpose. Further calculations are not yet possible, but it is a starting.

### 8.5 Measurement results and tolerances

There is a mocking proverb that says: "He who measures measures dung" - and the old master locksmith's saying: "Centimetres are a watchmaker's measure". Although the measurements in the above sections are intended to provide a basic understanding and make no claim to high accuracy, the following at least points out the sources of tolerances and their propagation.
Here is a simple example from electrical engineering: The measured voltage value across a resistor R may be subject to a tolerance $\Delta \mathrm{U}$, which may be caused by the tolerance of both the resistance $\Delta \mathrm{R}$ and the current $\Delta \mathrm{I}: ~ U \pm \Delta U=(I \pm \Delta I) \cdot(R \pm \Delta R)$. The value $\Delta \mathrm{U}$ is determined by partial differentiation according to the individual components, whereby the other components are considered constant:

$$
\begin{align*}
U & =I \cdot R \\
\Delta U & =\left|\frac{\partial U}{\partial I}\right| \cdot \Delta I+\left|\frac{\partial U}{\partial R}\right| \cdot \Delta R  \tag{8.4}\\
\Delta U & =R \cdot \Delta I+I \cdot \Delta R
\end{align*}
$$

It is useful to generate the relative sizes of the deviations from this

$$
\begin{align*}
\frac{\Delta U}{U} & =\frac{R \cdot \Delta I+I \cdot \Delta R}{I \cdot R} \\
\frac{\Delta U}{U} & =\frac{\Delta I}{I}+\frac{\Delta R}{R} \tag{8.5}
\end{align*}
$$

so that the relative tolerance TO in abbreviated form is

$$
T O_{U}=T O_{I}+T O_{R}
$$

So if, in an electronic circuit, the voltage drop across this resistance may only have a maximum tolerance of $\mathrm{TO}_{U} \leq 5 \%$ for functional reasons, then this resistor may only have a tolerance of $\mathrm{TO}_{\mathrm{R}}= \pm 2 \%$ if the current may also have an uncertainty of approximately $\mathrm{TO}_{\mathrm{I}}= \pm 2 \%$. The calculation procedure for the tolerance analysis of the speed measurement of the take-off trolley, see section 8.2 , is similarly simple.

Another example concerns the measurements of orbital and descent velocity, from which the coefficients $\mathrm{c}_{1}$, $\mathrm{c}_{\mathrm{d}}$ can be determined. The measurement technology for this is available today, it records these values and we have to insert them into the corresponding equations resolved according to $c_{l}$ and $c_{d}$ :

$$
\begin{equation*}
v=\sqrt{\frac{\mathrm{mass} \cdot g}{S_{w} \cdot \frac{\rho}{2}} \cdot \frac{1}{\sqrt{c_{l}{ }^{2}+{c_{d}}^{2}}}} \quad \quad v_{s i}=\sqrt{\frac{\operatorname{mass} \cdot g}{S_{w} \cdot \frac{\rho}{2}} \cdot \frac{c_{d}{ }^{2}}{\sqrt{\left(c_{l}{ }^{2}+c_{d}{ }^{2}\right)^{3}}}} \tag{5.4}
\end{equation*}
$$

each resolved according to $\mathrm{c}_{\mathrm{l}}\left(\mathrm{v}, \mathrm{v}_{\mathrm{sr}}\right)$ and $\mathrm{c}_{\mathrm{d}}\left(\mathrm{v}, \mathrm{v}_{\mathrm{sr}}\right)$ [La94]
$c_{l}=\frac{\operatorname{mass} \cdot g}{S_{w}} \cdot \frac{2}{\rho} \cdot \frac{1}{v^{2}} \cdot \sqrt{1-\left[\frac{v_{s r}}{v}\right]^{2}}$

$$
\begin{equation*}
c_{d}=\frac{\mathrm{mass} \cdot g}{S_{w}} \cdot \frac{2}{\rho} \cdot \frac{v_{s r}}{v^{3}} \tag{8.6a}
\end{equation*}
$$

How are the tolerances of the measured values of $v$ and $v_{\text {sr }}$ included in the accuracy of the coefficients? The calculation is somewhat more complex here. The result for the dependence of the lift coefficient and for the dependence of the drag coefficient is
$\frac{\Delta c_{l}}{c_{l}}=\frac{\Delta \rho}{\rho}+2 \cdot \frac{\Delta v}{v}+\frac{1}{\left(\frac{v}{v_{s r}}\right)^{2}-1} \cdot \frac{\Delta v_{s r}}{v_{s r}}$

$$
\begin{equation*}
\frac{\Delta c_{w}}{c_{w}}=\frac{\Delta \rho}{\rho}+3 \cdot \frac{\Delta v}{v}+\frac{\Delta v_{s r}}{v_{s r}} \tag{8.7a}
\end{equation*}
$$

or in short form

$$
\begin{equation*}
T O_{c_{l}}=T O_{\rho}+2 \cdot T O_{v}+\frac{T O_{v_{s r}}}{E^{2}-1} \tag{8.8a}
\end{equation*}
$$

$$
\begin{equation*}
T O_{c_{d}}=T O_{\rho}+3 \cdot T O_{v}+T O_{v_{s r}} \tag{8.8b}
\end{equation*}
$$



Fig 8.12 Speed polar of the RS3 F3J model [JW04], see also Fig 6.7a

We can see from this that the coefficients are particularly sensitive to the tolerance of the speed values: their measurement errors are included with factor of 2 for $c_{1}$ and a factor of 3 for $c_{d}$. With regard to the factor for the sink rate in $\mathrm{Eq}(8.7 \mathrm{a})$ or (8.8a), it should be noted that a range $\mathrm{E}<8$ is rarely of interest for polar measurements; thus the third term in $\mathrm{Eq}(8.8 \mathrm{a})$ usually falls by the wayside, see as an example the velocity polar of the RS3 in Figure 8.12. Even steeper trajectory inclination flights, as carried out for force measurements on deflected wing flaps, see Figure 8.14b, remained at $\mathrm{E} \geq 2$.4.

### 8.6 Servobelastung

Figure 5.3 already pointed out the advantage of variable airfoil geometries using flaps, especially in connection with the F3J-model RS3. In these cases, all wing sections are usually cambered at the same angle using the remote control setting. In addition, very large angles are also of interest, for example, if the model is to be brought down from a greater height without a significant amount of lift; the flaps then have the function of a brake and are not necessarily extended over the entire wing. It is easy to imagine that in these cases a greater force must be absorbed by the servo than with smaller angles.


Fig 8.13 Force sensor between servo and wingflap [Sc13].

The modelling industry offers servos in a wide torque range. But what values are involved in practice? In a Jugend-forscht project, theoretical estimates and measurements were made [Sc13].

Figure 8.13 shows the arrangement of the (tension-) force sensor between servo and a wing flap. The servo torque that must resist the torque
of the rudder flap can be specified for the sketched rudder position::

$$
\begin{equation*}
M_{\text {servo }}=F_{\text {sens }} \cdot h_{s}=F_{L u f t} \cdot \frac{h_{s} \cdot n \cdot l_{k l}}{h_{k l} \cdot \cos (\beta)} \tag{8.9}
\end{equation*}
$$

With the measured value of the path inclination flight shown in Figure $8.14 \mathrm{~b} \mathrm{~F}_{\text {sens }}=3 \mathrm{~N}$ and due to $\mathrm{hs}=$ $1 \mathrm{~cm}, \mathrm{M}_{\text {servo }}=3 \mathrm{Ncm}$ results. Assuming the main load is applied at $\mathrm{n}=0.5$, the air flow presses on the flap with $\mathrm{F}_{\text {air }} \approx 10 \mathrm{~N}$; the velocity is $\mathrm{v}=14.7 \mathrm{~m} / \mathrm{s}$,


Fig 8.14a Test model Arcus Sport/robbe and b) Measurement plot: a trajectory inclination flight [Sc13] Lower two curves: status signal "flap" and sensor signal $F_{\text {sens }}$; upper two curves: quasi-linear height reduction $h(t)$ and rectangular signal as response of the speed sensor $v(t)$; $v=d h / d t$

As a result of the inclination flights and confirmation measurements in the Dresden-Klotzsche wind tunnel, a flap drag coefficient $\mathrm{c}_{\mathrm{d}_{-} \mathrm{fl}} \approx 1.25$ was defined. This means that the air force acting on the flap can be calculated approximately:

$$
\begin{equation*}
F_{\text {air }}(\beta)=0,5 \cdot \rho \cdot c_{d_{-} f l} \cdot S_{f l} \cdot v^{2} \cdot \sin (\beta) \tag{8.10}
\end{equation*}
$$

Diagram Figure 8.15 shows force curves for the ArcusSport flap calculated using this formula. These corresponds well with the measured values.


Fig 8.15 Force $\mathrm{F}_{\text {air }}(\beta)$ acting on the arcus flap, calculated with $\mathrm{Gl}(8.10)$


The model is made of moulded foam (EPP). Its data are $\mathrm{b}=2.56 \mathrm{~m}, \mathrm{~S}_{\mathrm{w}}=51 \mathrm{dm}^{2}, \mathrm{~m}=2.3 \mathrm{~kg}$


$$
S_{f}=b_{f} \cdot l_{\mu f}=(4.5 \cdot 0,55) \cdot d m^{2}=2,5 \mathrm{dm}^{2}
$$

$$
\text { meas_exampl: } \mathrm{v}=14,7 \mathrm{~m} / \mathrm{s}, \mathrm{v}_{\mathrm{si}}=6,1 \mathrm{~m} / \mathrm{s}
$$

$$
(\mathrm{E}=2,4)
$$

$$
\beta=42^{0} \rightarrow \text { Force at servo } \mathrm{F}_{\mathrm{s}}=3 \mathrm{~N}
$$

Another example shows qualitatively the effect of centrifugal force on a flap, Figure 8.16. The model flies at a quasi-stationary speed $v$ until the time $t_{1}=3-49.014 \mathrm{Min}$, then the elevator is pressed to initiate the
loop. The speed $v$ decreases, the centrifugal force $\mathrm{F}_{\mathrm{c}_{-} \text {el }}\left(=\mathrm{Fz}_{\mathrm{HR}}\right)$ acting on the elevator increases until the loop is released at $\mathrm{t}_{2}=3-50.014$.


Fig 8.16 Inverted loop of an MPX Parkmaster 3D and measuring plot with centrifugal force FzHs
For larger aerobatic models, such as the Mx2, the effect of 3D figures on the servo load is now of interest. For this purpose, it is important to determine the position of the centre of gravity of the


Fig 8.17a The Mx 2 is designed for 3D aerobatics


Fig 8.17b Coordinates of the cg of aileron $\mathrm{xs}_{\mathrm{s}}=375 \mathrm{~mm} ; \mathrm{ys}_{\mathrm{s}}=76,4 \mathrm{~mm} ; \mathrm{Ms}_{\mathrm{s}}=\mathrm{maR}^{*} 83 \mathrm{~mm}$
control surfaces, see the aileron in Fig. 8.17b. If this centre of gravity is known, the hinge moment $\mathrm{MS}=\mathrm{m}_{\text {ail }} * 83 \mathrm{~mm}=1.16 \mathrm{kgcm}$ can be specified using the mass of the aileron, e.g. $\mathrm{m}_{\text {ail }}=140 \mathrm{~g}$. If the model is standing on the ground or flying horizontally, the gravitational acceleration $\mathrm{g}=$ $9.81 \mathrm{~m} / \mathrm{s}^{2}$ leads to the static rudder moment $\mathrm{M}_{\text {ail }}=11.4 \mathrm{Ncm}$. If manoeuvres are flown, the resulting rudder moments can be estimated and it can be checked whether the use of servos recommended by the model manufacturer is justified.

Center of gravity position:


$$
\begin{aligned}
& z_{0}=\frac{h}{3} * \frac{a+2 b}{a+b} \\
& z_{0}=\frac{h}{3} * \frac{2 a+b}{a+b}
\end{aligned}
$$

The center of gravity lies on the line connecting of the centers of the parallel sides

Fig 8.18 Graphical determination of the centre of gravity [Gerh]
Load cases for flaps are
*airflow pressure during flap deflection or in the spin motion
*forces that arise during acceleration processes

Fig 8.19 Control surfaces of the Mx 2 model and position of the individual centres of gravity

| contr. surfaces $\rightarrow$ | aileron | elevator |
| :---: | :---: | :---: |
| Flight manoeuvres | $\mathrm{R}_{\text {ail }}=83 \mathrm{~mm}, \mathrm{~m}_{\text {ail }}=140 \mathrm{gm}$ | $\mathrm{R}_{\mathrm{el}}=95 \mathrm{~mm}, \mathrm{~m}_{\mathrm{el}}=55 \mathrm{gm}$ |
| Model at rest or horizontal flight | $\begin{aligned} & \text { The acceleration due to gravity } \mathrm{g}=9,81 \mathrm{~m} / \mathrm{s}^{2} \text { generates a } \\ & \text { static flap moment with the distance } \mathrm{R}_{\text {sil }}=83 \mathrm{~mm} \text { of } \\ & \qquad \mathrm{M}_{\text {sill }}=11 \mathrm{Ncm} \end{aligned}$ | The acceleration due to gravity $\mathrm{g}=9,81 \mathrm{~m} / \mathrm{s}^{2}$ generates a static flap moment with the distance $\mathrm{Rhr}=93 \mathrm{~mm}$ $\mathrm{M}_{\mathrm{el} \_ \text {st }}=5 \mathrm{Ncm}$ |
| Landing strokes acceleration | $\begin{aligned} \text { assumptions: sink rate } \mathrm{v}_{\mathrm{sr}} & =1 \mathrm{~m} / \mathrm{s} ; \text { braking time } 0.1 \mathrm{~s} \\ & \rightarrow \text { flap moment } \quad \mathrm{M}_{\text {ail }} \mathrm{br}=23 \mathrm{Ncm} \end{aligned}$ |  |
| Looping acceleration | assumptions: $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$, radius of loop $\mathrm{r}=1 \mathrm{~m} \rightarrow$ flap moment $\quad \mathrm{M}_{\text {ail } \_10}=116 \mathrm{Ncm}$ | assumptions: $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$, <br> radius of loop $\mathrm{r}=1 \mathrm{~m} \rightarrow$ flap moment $\mathrm{M}_{\mathrm{el} \mathrm{\_} 10}=45 \mathrm{Ncm}$ |
| flap deflection air flow pressure | assumptions: $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$, <br> flap angle $\beta=45^{\circ} \quad \rightarrow$ flap moment $\quad \mathrm{M}_{\text {sil_ait }}=50 \mathrm{Ncm}$ | assumptions: $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$, <br> flap angle $\beta=45^{\circ} \quad \rightarrow \quad$ flap moment $\quad \mathrm{M}_{\text {el_ang }}=34 \mathrm{Ncm}$ |
| Spin <br> acceleration <br> air flow pressure <br> total <br> acceleration <br> air flow pressure <br> total |  | For the calculation, an as sumption is missing for the radius with which the elevator circles downwards around an imaginary line during the spin |
| Interception acceleration | $\begin{aligned} & \text { assumed: airfoil NACA } 0012 ; \mathrm{c}_{\max }=0,8 ; \mathrm{c}_{\mathrm{d}}=0,04 ; \\ & \text { mass }=3 \mathrm{~kg} ; \\ & \begin{aligned} \mathrm{v}=50 \mathrm{~m} / \mathrm{s} ; \text { Radius } \mathrm{r} & =10 \mathrm{~m}: \rightarrow \text { load multible } 1 \mathrm{v}=20 \\ & \rightarrow \text { flap moment } \quad \mathrm{M}_{\text {sil_int }}=230 \mathrm{Ncm} \end{aligned} \end{aligned}$ | load multible $1 \mathrm{v}=20$ <br> $\rightarrow$ flap moment $\quad \mathrm{M}_{\text {el_ }{ }_{\text {int }}=102 \mathrm{Ncm}}$ |

Tab. 8.1 Flight manoeuvres of a 3D aerobatic model and expected flap moments

### 8.7 Solar drive - how many cells are required?

In contrast to man-carrying aircraft, such as the Solarimpuls_2 [SI], in the model, the wing alone is considered as a surface for solar cells. For the sake of simplicity, a rectangular wing is assumed; design (strength) reasons mean that only a wing area reduced by a factor of $k$ is available as a surface for the solar cells (red outline). Two typical values are assumed:
*k1 $=0,67$ : the solar cells cover from the main spar to the trailing edge;

* $\mathrm{k} 2=0,88$ : the solar cells cover from the leading edge to the trailing edge.


Fig 8.20 Half-span of a rectangular wing
The question of the number of cells means: How large must the span be so that the model can climb safely? To do this, we need to know how much heavier the model will be when the solar cells are installed. Figure 7.3 shows the statistically determined relationship between the model mass m and the wing area $\mathrm{S}_{\mathrm{w}}$ of motorised gliders built in the Modellflugclub Rossendorf without solar cells. This resulted in the function for the regression line

$$
\begin{equation*}
x\left(S_{w}\right)=0,19 \mathrm{~kg}+0,024 \frac{\mathrm{~kg}}{\mathrm{dm}^{2}} \cdot S_{w} \tag{8.11}
\end{equation*}
$$

As an example of solar cells, we take those from Lemo-Solar [LeSol]. Their specific mass is $\mathrm{B}_{\mathrm{SC}}=$ $(20 \ldots 26,6) \mathrm{g} / \mathrm{dm}^{2}$. This results in the new function for the mass m versus the wing area

$$
\begin{equation*}
m\left(S_{w}\right)=x\left(S_{w}\right)+B_{S C} \cdot k \cdot S_{w} \tag{8.12}
\end{equation*}
$$

The propeller must deliver at least the power according to $\mathrm{Eq}(5.11)$. Taking into account the efficiency of the actuator, motor and propeller, the electrical power is therefore required from the solar cells:

$$
\begin{equation*}
P_{S C_{-} \text {req }}\left(S_{w}\right) \geq \frac{m\left(S_{w}\right) \cdot g \cdot\left[v_{s r}\left(S_{w}, m\right)+v_{c r}\right]}{\eta_{c t r} \cdot \eta_{\text {mot }} \cdot \eta_{\text {prop }}} \tag{5.11a}
\end{equation*}
$$

However, the rate of descent also depends on the model mass $\mathrm{m}\left(\mathrm{S}_{\mathrm{w}}\right)$ and wing area $\mathrm{S}_{\mathrm{w}}$ :

$$
\begin{equation*}
v_{s i}\left(S_{w}\right)=\sqrt{\frac{m\left(S_{w}\right) \cdot g}{S_{w} \cdot \frac{\rho}{2}} \cdot \frac{\left(c_{d p}+\frac{c_{l}{ }^{2}}{\pi \cdot S_{w}} \cdot l_{\mu}^{2}+c_{d h}\right)^{2}}{\sqrt{\left(c_{l}{ }^{2}+\left(c_{d p}+\frac{c_{l}}{\pi \cdot S_{w}} \cdot l_{\mu}^{2}+c_{d h}\right)^{2}\right)^{3}}}} \tag{5.4a}
\end{equation*}
$$

Using $\operatorname{Eq}(5.4 a)$ and $\operatorname{Eq}(8.12)$ in $\operatorname{Eq}(5.11 a)$, we obtain the power $\mathrm{P}_{\text {SC_requ }}$ required by the solar cells as a function of the wing area $\mathrm{S}_{\mathrm{w}}$, Figure 8.21, red curve.


Fig 8.21 Requested and delivered power of the
Solar cells with coverage k2 $=0.88$

The following efficiency levels were assumed $\eta_{\text {st }}=0,95 ; \eta_{\text {mot }}=0,85$ und $\eta_{\text {prop }}=0,6$; for cells we use $B_{s c}=20 \mathrm{~g} / \mathrm{dm}^{2}$.

The constant $1000 \mathrm{~W} / \mathrm{m}^{2}$ applies to solar cells in space; due to atmospheric attenuation, $800 \mathrm{~W} / \mathrm{m}^{2}$ is calculated on earth. The efficiency is usually $\eta_{\mathrm{SC}} \approx$ 0.15 . This means that the power delivered by the cells can be written as follows

$$
\begin{equation*}
P_{S C_{-} d e l}\left(A_{f}\right)=k \cdot A_{f} \cdot 800 \cdot \frac{W}{m^{2}} \cdot \eta_{S C} \tag{8.13}
\end{equation*}
$$

The blue curve in Figure 8.21 shows this function. At the intersection point at $S_{w} \approx 43 \mathrm{dm}^{2}$ the cell power just compensates the sink rate, the model flies horizontally. With a wing area $\mathrm{S}_{\mathrm{w}} \approx 75 \mathrm{dm}^{2}$ the cell power generates $\mathrm{P}_{\mathrm{SC} \_ \text {del }} \approx 80 \mathrm{~W}$ and thus a climb speed of $\mathrm{v}_{\mathrm{cr}}=0.7 \mathrm{~m} / \mathrm{s}$.
There is no room for manoeuvre for higher climb rates, e.g. reducing the flight mass from $m\left(75 \mathrm{dm}^{2}\right)=3.3 \mathrm{~kg}$ $\rightarrow 2.5 \mathrm{~kg}$ would only result in an improvement to $\mathrm{v}_{\mathrm{cr}} \approx 1 \mathrm{~m} / \mathrm{s}$.

In practice, the model could be realised with a wingspan $b=3 \mathrm{~m}$ and a chord length of $1 \mu=250 \mathrm{~mm}$ with 10 solar cells per half-span, each supplying around 12 Vx 7 A . As battery 3 S LiPo could be used for storage.
8.8 Start process of a model, calculated with the help of various precise mathematical approaches
8.8.1 Velocity $\mathrm{v}(\mathrm{t})$ and take-off roll distance $\mathrm{s}(\mathrm{t})$ without air resistance and rolling friction

The force F accelerating the model is equal to the thrust force $\mathrm{F}_{\mathrm{T}}$ :

$$
\begin{gathered}
F=F_{T} \\
m * a=F_{T} \rightarrow a=F_{T} / m
\end{gathered}
$$

For a motion process according to Fig 8.22 yields

$$
\mathrm{v}(\mathrm{t})=\mathrm{a}^{\star} \mathrm{t}+\mathrm{v}_{0},
$$

and mit $\mathrm{v}_{0}=0$ the speed curve is


Fig 8.22 Velocity curve without resistances

$$
\begin{equation*}
v(t)=\frac{F_{T}}{\text { mass }} \cdot t \tag{8.14}
\end{equation*}
$$

Accordingly, the speed $v_{1}$ is reached at the time $t=t_{1}$. With uniform acceleration, i.e. $a=$ const, the distance $\mathrm{s}(\mathrm{t})$ results from the mean value of both velocities:

$$
\begin{equation*}
s(t)=\frac{v_{0}+v_{1}}{2} \cdot t \tag{8.15}
\end{equation*}
$$

and with $\mathrm{v}_{0}=0: \quad s\left(t_{1}\right)=\frac{v_{1}}{2} \cdot t_{1}$
Aus $\mathrm{Gl}(8.15): \quad t_{1}=\frac{\text { mass }}{F_{T}} \cdot v_{1}$
in $\mathrm{Gl}(8.15): \quad s\left(t_{1}\right)=\frac{v_{1}{ }^{2}}{2} \cdot \frac{\text { mass }}{F_{T}}$
example: mass $m=3 \mathrm{~kg}, \mathrm{~F}_{\mathrm{T}}=6 \mathrm{~N}$. After what roll time $\mathrm{t}_{\mathrm{lo}}$ does the model reach take-off velocity $\mathrm{v}_{\mathrm{lo}}=10 \mathrm{~m} / \mathrm{s}$ und how long is take-off distance?

$$
\begin{aligned}
& \text { mit } \mathrm{t}_{1}=\mathrm{t}_{\text {abh }} \text { und } \mathrm{v}\left(\mathrm{t}_{1}\right)=\mathrm{v}\left(\mathrm{t}_{\text {abh }}\right) \text { wird } \\
& \text { nach } \mathrm{Gl}(8.14 \mathrm{a}) \quad t_{a b h}=v_{a b h} * \frac{\text { mass }}{F_{T}}=10 \cdot \frac{\mathrm{~m}}{\mathrm{~s}} \cdot \frac{3 \cdot \mathrm{~kg}}{6 \cdot \mathrm{~N}}=10 \cdot \frac{\mathrm{~m}}{\mathrm{~s}} \cdot \frac{3 \cdot \mathrm{~kg}}{6 \cdot \mathrm{~kg} \cdot \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=5 \mathrm{~s} \\
& \text { nach } \mathrm{Gl}(8.16) \quad s\left(t_{a b h}\right)=\frac{v_{a b h}^{2}}{2} \frac{\mathrm{mass}}{F_{T}}=\frac{\left(10 \cdot \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{2} \cdot \frac{3 \cdot \mathrm{~kg}}{6 \cdot \mathrm{~N}}=50 \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}} \cdot \frac{1 \cdot \mathrm{~kg}}{2 \cdot \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}}=25 \cdot \mathrm{~m}
\end{aligned}
$$

8.8.2 Speed $v(t)$ and take-off roll distance $s(t)$ taking into account the air resistance

The force F accelerating the model is now equal to the value obtained by subtracting the braking drag force $\mathrm{F}_{\mathrm{D}}$ from the thrust force $\mathrm{F}_{\mathrm{T}}$ :

$$
\mathrm{F}=\mathrm{F}_{\mathrm{T}}-\mathrm{F}_{\mathrm{D}}
$$

If the speed $v(t)$ has become so great that the drag force FD has reached the magnitude of the thrust force, i.e. $\mathrm{F}_{\mathrm{D}}=\mathrm{F}_{\mathrm{T}}$, an increase in speed is no longer possible.

From

$$
F_{D}=\frac{\rho}{2} \cdot v^{2} \cdot c_{d} \cdot S
$$

folgt

$$
v=\sqrt{F_{D} \cdot \frac{2}{\rho} \cdot \frac{1}{c_{d}} \cdot \frac{1}{S}}
$$

und mit $F_{D}=F_{T}$

$$
\begin{equation*}
v_{\max }=\sqrt{F_{T} \cdot \frac{2}{\rho} \cdot \frac{1}{c_{d}} \cdot \frac{1}{S}} \tag{8.17}
\end{equation*}
$$

example: $\mathrm{S}=60 \mathrm{dm}^{2}=0.6 \mathrm{~m}^{2}, \mathrm{~F}_{\mathrm{T}}=6 \mathrm{~N}$, aerodynamic quality $\mathrm{E}=10$, lift coefficient $\mathrm{c}_{1}=0,8$; drag coefficient $\mathrm{c}_{\mathrm{d}}=\mathrm{c} / \mathrm{E}=0,08$. What maximum speed $\mathrm{v}_{\text {max }} \mathrm{can}$ be achieved with the thrust $\mathrm{F}_{\mathrm{T}}$ ?
nach Gl (8.17)

$$
\begin{aligned}
& v_{\max }=\sqrt{6 \cdot \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}} \cdot \frac{2}{1.25 \cdot \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \cdot \frac{1}{0,08}} \cdot \frac{1}{0.6 \cdot \mathrm{~m}^{2}}} \\
& v_{\max }=14.14 \cdot \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

However, it is not possible to make a statement about the velocity curve, such as according to Eq (8.14). For this, the process must be described as a differential equation (DGL):

$$
\begin{align*}
m \cdot \frac{d v}{d t} & =F_{T}-F_{D} \\
\frac{d v}{d t} & =\frac{F_{T}}{m}-\frac{1}{m} \cdot \frac{\rho}{2} \cdot v^{2} \cdot c_{d} \cdot S \tag{8.18}
\end{align*}
$$

The required solution to this differential equation concerns the velocity $\mathrm{v}(\mathrm{t})$. Due to its non-linear cha-racter $-\mathrm{v}(\mathrm{t})$ occurs in the square - this can only be approximated analytically. Modelling the equation in a suitable calculation program, here Matlab\&Simulink, is more detailed and also more elegant:


Fig 8.23 Simulink solution of the $\operatorname{DGL}(8.18)$. The velocity $v(t)$ reaches its maximum at $t \approx 20$ s
The differential equation is described as a mathematical model at the top left. At the input of the first integrator, think of the derivative $\mathrm{dv} / \mathrm{dt}$. Then the quantity $\mathrm{v}(\mathrm{t})$ is obtained at its output. The velocity must be squared (block product) and multiplied by the corresponding coefficients $1 / \mathrm{m}, \rho / 2, \mathrm{cw}, \mathrm{S}$ and fed back to the input. The values of these coefficients are contained in the Gain block. At the top left, the constant is realised $\mathrm{F}_{\mathrm{T}} / \mathrm{m}$. The second integrator generates the path $\mathrm{s}(\mathrm{t})$ from the velocity $\mathrm{v}(\mathrm{t})=\mathrm{ds} / \mathrm{dt}$.

The solution shows the time course of velocity $v(t)$, purple curve, and $s(t)$, yellow curve. You can see that $\mathrm{v}_{\mathrm{abh}}=10 \mathrm{~m} / \mathrm{s}$ is reached after $\mathrm{t}_{\mathrm{abh}}=6 \mathrm{~s}$, but now with a slightly longer rolling distance than calculated according to Eq (8.16), namely at $\mathrm{s} \approx 30 \mathrm{~m}$. From this point onwards, the path should no longer be shown, as the model is now flying. The model reaches the maximum speed of $v_{\text {max }} \approx 14 \mathrm{~m} / \mathrm{s}$ according to $\mathrm{Eq}(8.16)$ in $\mathrm{t} \approx 20 \mathrm{~s}$, after it has already been in the air for 14 s .
8.8.3 Velocity $\mathrm{v}(\mathrm{t})$ and take-off roll distance $\mathrm{s}(\mathrm{t})$, taking into account air resistance and rolling friction

The rolling friction $\mathrm{F}_{\mathrm{RF}}$ must now be subtracted from the thrust force in addition to the drag force. The force $F$ accelerating the model is then
where

$$
\mathrm{F}=\mathrm{F}_{\mathrm{T}}-\mathrm{F}_{\mathrm{D}}-\mathrm{F}_{\mathrm{RF}}
$$

$$
\mathrm{F}_{\mathrm{RF}}=\mu\left(\mathrm{F}_{\mathrm{W}}-\mathrm{F}_{\mathrm{L}}\right)
$$

$$
F_{R F}=\mu \cdot\left(m \cdot g-\frac{\rho}{2} \cdot v^{2} \cdot c_{l} \cdot S\right)
$$

The approach for the rolling friction force FR takes into account the fact that the model loads the ground less and less with increasing speed $v$ - corresponding to increasing lift. In the literature, a typical value is $\mu \approx 0.06$. In extension of (8.18):

$$
\begin{equation*}
\frac{d v}{d t}=\frac{F_{T}}{m}-\frac{1}{m} \cdot \frac{\rho}{2} \cdot v^{2} \cdot c_{d} \cdot S-\mu \cdot\left(g-\frac{\rho}{2} \cdot v^{2} \cdot c_{l} \cdot \frac{S}{m}\right) \tag{8.19}
\end{equation*}
$$

When modelling the DGL, it is important that the influence of rolling friction is set to zero when the starting speed vabh is reached - an option that can be implemented very elegantly in the simulation using a comparator function:


Fig 8.24 Simulink solution of the $\operatorname{DGL}(8.18)$. The lift-off speed of $10 \mathrm{~m} / \mathrm{s}$ is now reached at $t_{a b h}=8 \mathrm{~s}$, the rolling distance is $s=40 \mathrm{~m}$. At this point, the influence of rolling friction is switched off.

Compared to the solution without rolling friction, Figure 8.23, the time of lift-off speed is extended to $t_{a b h}=8 \mathrm{~s}$, the rolling distance is now about 40 m . A comparator circuit Switch switches off the "rolling friction" signal path at the time $t_{a b h}$ because the variable $\mu\left(F_{W}-F_{L}\right)$, green curve, reaches the value zero.

## What we can learn from it

The calculation of motion processes can be carried out with varying degrees of accuracy in the description of the physical facts. Using the take-off process of a model aircraft as an example, it is shown that the basic equations are sufficient for an initial estimate of the roll time and roll distance.

If the course of time and starting distance are of more interest, this requires a more refined description in the form of differential equations. In most cases, their solution is only possible with the help of common simulation methods.

|  | Basic equations | Consideration of air <br> resistance | Consideration of air resistance <br> and rolling friction |
| :--- | :--- | :--- | :--- |
| $\mathrm{V}_{\text {abh }}$ | $10 \mathrm{~m} / \mathrm{s}$ | $10 \mathrm{~m} / \mathrm{s}$ | $10 \mathrm{~m} / \mathrm{s}$ |
| $\mathrm{t}_{\text {abh }}$ | 5 s | 6 s | 8 s |
| $\mathrm{~S}_{\text {abh }}$ | 25 m | 30 m | 40 m |
| $\mathrm{~V}_{\text {max }}$ | $14 \mathrm{~m} / \mathrm{s}$ | $14 \mathrm{~m} / \mathrm{s}$ | $14 \mathrm{~m} / \mathrm{s}$ |

Tabelle 8.2: Take-off values of a flight model calculated according to different mathematical approaches

### 8.9 Class F3K - Is it worth building lightweight models?

In the model flying class, the question is being discussed as to whether the greater height achieved by a heavier model does not also ensure a longer glide? [F3K-For]?



Fig 8.25a): Hand-launch of a F3K-model: world champion 2019, Henri Sander; Foto: Izabela Krawczyk [Kra19]; b) Splitting the oblique throw into vectors [uni_heidbg20]

The forces acting on a body with friction when thrown at an angle are described by a system of differential equations in vector form [uni_heidbg20].

The initial velocity acts
in x-direction with $\quad v_{o} \cdot \cos (\varphi) \quad \frac{d x}{d t}=v_{o} \cdot \cos (\varphi)$
in y-direction with $\quad v_{o} \cdot \sin (\varphi) \quad \frac{d y}{d t}=v_{o} \cdot \sin (\varphi)$
In ananolgy to $\mathrm{Gl}(8.19)$ it yields
in x-direction: $m \cdot \frac{d^{2} x}{d t^{2}}=-\frac{\rho}{2} \cdot c_{w} \cdot A \cdot \cos (\varphi) \cdot \frac{d x}{d t} \cdot \sqrt{\left(\frac{d x}{d t}\right)^{2}+\left(\frac{d y}{d t}\right)^{2}} \quad \frac{d x}{d t}(0)=v_{o} \cdot \cos (\varphi)$
in y-direction: $\quad m \cdot \frac{d^{2} y}{d t^{2}}=-\frac{\rho}{2} \cdot c_{w} \cdot A \cdot \sin (\varphi) \cdot \frac{d y}{d t} \cdot \sqrt{\left(\frac{d x}{d t}\right)^{2}+\left(\frac{d y}{d t}\right)^{2}}-m \cdot g \quad \frac{d y}{d t}(0)=v_{o} \cdot \sin (\varphi)$

Using the Simulink programme applicable to these equations, Figure 8.26, the spin start is calculated with the following parameters:
initial speed $\mathrm{v}_{\mathrm{o}}=25 \mathrm{~m} / \mathrm{s}$; launch angle $\varphi=70^{\circ}$
mass $=100 \mathrm{~g}, 150 \mathrm{~g}, 200 \mathrm{~g} ;$ drag coefficient $\mathrm{c}_{\mathrm{d}}: 0.01,0.02$;
From the maximum altitude reached, criterion $v_{y}(t) \rightarrow 0$, the duration $t g$ for the glide is calculated with $\operatorname{tg}_{\max }=\mathrm{y}(\mathrm{t})_{\max } / \mathrm{v}_{\mathrm{si}}$ and the sink rate $\mathrm{v}_{\mathrm{si}}$ according to $\mathrm{Eq}(5.4)$ with the parameters mass $=100 \mathrm{~g}, 150 \mathrm{~g}, 200 \mathrm{~g} ;$ link coefficient $\mathrm{c}_{1}=1,0 ;$ drag coefficient $\mathrm{c}_{\mathrm{d}}=\mathrm{c}_{1} / 10$.


Fig 8.26 Si Simulink programme for the differential equation system $\operatorname{Gln}(8.20 a, b)$


Fig 8.27 Solution curves of the DGL system Eq(8.20a,b) and numerical results for the sliding time tg/s

## What can we learn from this

* with increasing mass, the achievable take-off altitude $\mathrm{y}(\mathrm{t})_{\text {max }}$ becomes greater, Figure 8.27; with the same initial speed vo $=25 \mathrm{~m} / \mathrm{s}$, the heavier model compensates better for the drag force $\left(\sim \mathrm{v}^{2}\right)$, which is particularly effective at the beginning of flight.
* With increasing mass, the rate of descent increases to such an extent that the height gain is cancelled out, the flight time in gliding flight becomes shorter.
* Lightweight construction is worthwhile!

Submission of extracurricular cooperation partners for schools with all-day programmes (GTA)

Title of the offer/project/AG: Aeromodelling

Provider/name of cooperation partner:
(Adresse, Tel., Fax, Email)

Modellflugclub Rossendorf e.V.<br>Hans Langenhagen<br>Bautzner Landstraße 26<br>01456 Radeberg OT Rossendorf<br>mfcr@fz-rossendorf.de

Objectives: What should be achieved with the offer?
Model airplane construction: acquire the basics of craftsmanship, learn to read drawings; understand the load-appropriate use of materials;
Model flight: understanding the basics of flight physics, learning to control a model aircraft.
Internal impact: supporting physics lessons; external impact: publicising the grammar school through publications in newspapers and on the mfcR home page

Contents: What exactly will be done?
(Content, scheduling, involvement of students, requirements, age group, etc.)
Model airplane construction in the Rossendorf workshop and model flight in the gdb sports hall alternate on a weekly basis; in good weather conditions also flying on the mfcR club airfield (southern edge of Weißig).
1st course year: In the workshop, each student first builds a simple polystyrene and balsa glider, then groups of students build small remote-controlled motorised models (park flyers). In the sports hall, the students learn to control light - and therefore slow-flying - models (slow flyers); this is done using model flight simulators, teacher/student remote control systems and the club's own models.
2nd course year: depending on their interests, groups of students build a larger model (motorised glider) for use on the airfield and two slow flyers for indoor use. In terms of flying, they are offered the opportunity to take part in club comparison flights with motorised gliders, and they can practise simple aerobatics in the gym. Flight physics is also taught.
3rd course year: Design, partial construction and assembly of a model aircraft by groups of students. The flying tests should also be carried out with regard to the parameters achieved if possible (use of measurement technology)
4th course year: working on projects (e.g. minimum speed; maximum load ...)
Necessary material expenses: What is needed for the implementation? What does it cost?
(list in detail, indicate prices) for SY 2012_13
Material costs $\quad 874 €$
Labour costs $\quad 700 €$
Necessary labour expenses: What hourly remuneration is necessary? $17,50 €$

## Specify criteria for the labour costs:

- Qualification, degree, certificates or similar. Dipl.-Ing. of a technical specialisation
- Preparation effort and scope $\quad \geq 5 \mathrm{~h} /$ event
- Pedagogical suitability, references if applicable
- Personal suitability
- Group size/how many pupils can take part, what age group? $\leq 15$ ( 6 to 9 deg )

The all-day programme (GTA)) Model flying was offered at the Dresden-Bühlau secondary school (gdb) for over 10 years: from September 2009 to July 2019. Rudolf Zimmermann and Hans Langenhagen, who were responsible for the programme, started this working group in the hope of strengthening the club's youth membership, which was weakening at the time. The aim was also to introduce pupils to aviation topics that could be linked to their school physics and mathematics through an interesting programme.


In fact, the number of members of the youth group increased by around 10 pupils during the GTA period; the number of pupils joining from the lower classes and leaving from the upper classes was roughly balanced. However, there were also "drag" effects: Neighbouring children or friends who did not attend the high school joined.
Students with a longer-term interest remained in the working group even after the third year of the course; they now also built cleanly and purposefully and enjoyed taking part in comparison flights. The next few years will show how many of them grow up in the post_GTA_period.


As part of an all-day programme, the school requires two hours of supervision per week. As we had up to 10 interested students in the early days, we divided them up and alternated between model airplane construction in our club workshop and model flight - remote control - in the high school sports hall. This alone required a minimum of four supervisors; at the beginning, other club members were also very willing to help. In addition to motorised gliders from Höllein/Grüner construction kits, F. Schade also built hand-launch gliders (class F3K). The latter involved a self-built construction using balsa sheets as wings, on which the airfoil AG04 is milled. The company Aigner offers such Material. The mfcR also
continued to support the purchase of performance models in this class, which benefited interested pupils including his son Paul when he was promoted to the national team/youth class.

The indoor remote control training proved to be very effective [La16]. For example, the 2012/13 school year showed that four of the five beginners were able to take part in the Lilienthal competition at the end of the school year in the Steutz holiday week after a short training period in both the hand-lunch class and the motorised glider class.


From around 2016, A. Scheffler's offer to provide advanced students with indoor aerobatics training proved to be interesting and attractive. Between three and five of them took up this offer with enthusiasm. However, only a few of them came to the Margon Arena, where Alfons organised flying time for club members and external guests on several Sundays.


At the beginning of our GTA years, the grammar school had only reached about half of its planned pupil capacity. It was no problem to get two school hours for training in the hall. This was then reduced to one school lesson, where there were more frequent blockages due to overruns in the sports lessons ahead of us. It became obvious that the sports teachers wanted us out of the hall. The official reason given was that the number of pupils had grown to 1000 . Unofficially, we heard comments that our 3 to 5 pupils in a three-field hall ( 50 metres) would be considered a huge disproportion to the possible three school classes that could use the hall at the same time. We were granted alternative lessons at the weekend, but they were nowhere near as popular with the pupils as the lessons immediately after school.

But the club's capacity for coaching also dwindled, and from 2018 R. Zimmermann and I were once again coaching alone in the workshop. I moved the flying to our field: partly on the afternoons of the previous indoor times, partly on Saturdays, or I invited people to take part in competitions.


## What we can learn from this

Offering model flying as part of an all-day programme (GTA) is definitely the most direct way to bring children/young people into contact with this subject. However, a club should have sufficient staffing potential to ensure that it can also look after the "flash in the pan" candidates for the minimum required first half of the school year while providing the same level of interesting support for the advanced students.

The idea of introducing pupils to aviation topics that could be combined with their knowledge of physics and maths was only realised in a few cases, see the previous sections 8.1 to 8.4. The topic of "take-off trolley" in particular could have been expanded with regard to investigating the measurement method for variable angles of attack and aspect ratios.

Incidentally, measurements on topics 8.1 and 8.2 produced the acceleration value a $\square 2.3 \mathrm{~m} / \mathrm{s} 2$ for drives of very different sizes .... Does this reflect the starting behaviour of brushless motors? Here too, comparative measurements with larger models would be interesting.

This publication is an attempt to describe the supervision of model flying offered to pupils at a grammar school during a 10 -year period and to provide a theoretical basis for the sub-disciplines covered. Perhaps participants in this study group will not only be able to reminisce but also be inspired to deal with theoretical and metrological questions of any kind at a later date.
It is also possible, however, that there will be suggestions for aeromodelling colleagues who are thinking of setting up a student support programme in model flying.
Hans Langenhagen
Rossendorf, am 21. Juli 2021

## Acknowledgements

On behalf of the students, I would like to thank all my fellow members of the association who have contributed to the supervision over the years. They have not only contributed to the transfer of knowledge and education of young people, but have also helped to increase the number of members of the association. I would especially like to thank Rudi Zimmermann - unfortunately, he did not live to see the completion of this brochure - and to Alfons Scheffler for his direct support. I would like to thank Bedrich Pavka for his advice on tailplane design and Jiri Tomkovic for helpful discussions on dfferential equations. I would also like to thank Norbert Brückner for his valuable advice on propeller calculations and for taking the trouble to proofread the book.

### 9.0 Anhang

### 9.1 Calculation of torque replacement wing

The rectangular aerofoil geometry represents a special case of the possible wing shapes. However, this offers the advantage of a clear


Fig 9.1: The size and position of the replacement rectangular wing are calculated from the real trapezoidal ground plan of the aerofoil aerodynamic view, e.g. for finding the aerodynamic center (neutral point $\mathrm{NP}_{\mathrm{w}}$ ). The reference wing chord $1 \mu$ is determined for the real wing, shown as a line in Fig. 9.1, and the entire rectangular replacement wing is finally determined from the surface. Its neutral point is as known at $\mathrm{X}_{\mathrm{Nw}}=0.25^{*} \mid \mu$

JUST [Ju65] gives the following calculation rule for the double trapezoidal wing, Fig 9.2:
Wing surface $\quad S_{w}=s_{1} \cdot\left(l_{1}+l_{2}\right)+s_{2} \cdot\left(l_{2}+l_{3}\right)$
Reference wing chord: $l_{\mu}=\frac{2}{3 \cdot S_{w}} \cdot\left[S_{1}\left(l_{1}{ }^{2}+l_{2}{ }^{2}+l_{1} \cdot l_{2}\right)+s_{2} \cdot\left(l_{2}{ }^{2}+l_{3}{ }^{2}+l_{2} \cdot l_{3}\right)\right]$

Position of the replacement wing:

$$
\begin{equation*}
x_{E}=\frac{1}{3 \cdot S_{w}} \cdot\left[x_{2} \cdot s_{1}\left(l_{1}+2 \cdot l_{2}\right)+x_{2} \cdot s_{2} \cdot\left(l_{3}+2 \cdot l_{2}\right)+x_{3} \cdot s_{2} \cdot\left(l_{2}+2 \cdot l_{3}\right)\right] \tag{9.3}
\end{equation*}
$$



Fig 9.2: Zur Ermittlung des Rechteck-Ersatzflügels aus dem Doppel-Trapezflügel

Example 9.1
Where is the neutral point $\mathrm{x}_{\mathrm{Nw}}$ of the Strolch trapezoidal wing with the following dimensions, see Fig 7.1a:
$\mathrm{s}_{1}=3.5 \mathrm{dm}, \mathrm{s}_{2}=2.1 \mathrm{dm}, 1_{1}=1.6 \mathrm{dm}$, $1_{2}=1.6 \mathrm{dm}, 1_{3}=1.0 \mathrm{dm}$, $\mathrm{x}_{2}=0.0 \mathrm{dm}, \mathrm{x}_{3}=0.154 \mathrm{dm}$

First, we determine the position and size of the (rectangular) moment-replacement wing using the equations Eqs. 9.1 to (9.3). Based on this, we obtain the new centre point position of the trapezoidal wing:
according to $\operatorname{Eq}(9.1): \quad S_{w}=3,5 \mathrm{dm} \cdot(1,6 \mathrm{dm}+1,6 \mathrm{dm})+2,1 \mathrm{dm} \cdot(1,6 \mathrm{dm}+1,0 \mathrm{dm})=16,7 \mathrm{dm}^{2}$
according to $\mathrm{Eq}(9.2)$ :
$l_{\mu}=\frac{2}{3 \cdot 16,7 \mathrm{dm}^{2}} \cdot\left[3,5 \mathrm{dm}\left(1,6 \mathrm{dm}^{2}+1,6 \mathrm{dm}^{2}+2,56 \mathrm{dm}^{2}\right)+2,1 \mathrm{dm} \cdot\left(1,6 \mathrm{dm}{ }^{2}+1,0 \mathrm{dm}^{2}+1,6 \mathrm{dm}^{2}\right)\right]=1,51 \mathrm{dm}$
according to $\operatorname{Eq}(9.3)$ where $\mathrm{x}_{2}=0$ :
$x_{E}=\frac{1}{3 \cdot 16,7 \mathrm{dm}^{2}} \cdot\left[15,4 \cdot 2.1 \mathrm{dm}^{2} \cdot(1,6 \mathrm{dm}+2 \cdot 1 \mathrm{dm})\right]=2,33 \mathrm{~mm}$
The neutral point $\mathrm{x}_{\mathrm{Nw}}$ of the rectangular wing is at $\mathrm{x}_{\mathrm{Nf} \mathrm{R}}=0,25 * 1_{\mu}=0,25^{*} 1,51 \mathrm{dm}=0,377 \mathrm{dm}=37,7 \mathrm{~mm}$ from its leading edge. This means that the neutral point of the trapezoidal wing is 40 mm away from its leading edge $\mathrm{x}_{\mathrm{Nw}}=\mathrm{x}_{\mathrm{Nw}_{-} \mathrm{R}}+\mathrm{x}_{\mathrm{E}}=37,7 \mathrm{~mm}+2,33 \mathrm{~mm}=40 \mathrm{~mm}$. The measure for $\mathrm{x}_{\mathrm{Nf}}$ can then be used to determine the neutral point of the overall model $\mathrm{x}_{\mathrm{N}}$ (aerodynamic center of whole model) according to equations $\operatorname{Eqn}(7.5 \mathrm{a})$ to $(7.5 \mathrm{c})$ and Fig 7.12 . From this, the position of the centre of gravity $\mathrm{x}_{\mathrm{S}}$ is finally calculated for a given stability measure, see $\operatorname{Eq}(7.6 \mathrm{~b})$.

## Literatur

| [A80] | Althaus, D. <br> Profilpolaren für den Modellflug <br> Neckar-Verlag VS-Villingen, 1980, ISBN 3-7883-0158-9 |
| :---: | :---: |
| [A320_a] | Morgenstern/Plath <br> Airbus A330/A340 - Die großen Europäer <br> Motorbuchverlag Stuttgart, 1996 |
| [A320_b] | http://de.wikipedia.org/wiki/Airbus-A320-Familie\#Zwischenf.C3.A4Ile > Technische Daten geprüft: Mai 2014 |
| [A320_c] | http://www.aero.de/forum/Allgemeines-aus-der-Luftfahrtbranche/Gleitzahl-eines-Airbusses-A320-/ > clean configuration |
| [Aer19] | Leistungsdiagamm für Elektro-Luftschrauben Cam-Carbon, Grafik im Firmenkatalog Aeronaut 2019 |
| [Alt20] | https://www.aerobtec.com gepr. 2019 |
| [AS19] | Scheffler, A., persönliche Mitteilung 2019 |
| [BB14] | Böswirth, L. und S. Bschorer <br> Technische Strömungslehre <br> 10. überarbeitete und erweiterte Auflage 2014, Springer Vieweg |
| [BauH2/20] | Baumeister, W. <br> Thermik $\geq 0$ - Django Neon XL von Sansibar FMT_H02/2020, S. $74 \ldots 81$ |
| [BauH8/20] | Baumeister, W. <br> Jäger der verlorenen Thermik - Stratus 4000 von Schmierer/Valenta $\text { FMT_H08/2020, S. } 98 \ldots 105$ |
| [Beu58] | Beuermann, K. <br> Wie erreiche ich eine gute Längsstabilität Der Flugmodellbau, H.1, H2 1959 C. Lange Verlag Duisburg |
| [Böl90] | Bölkow, L. <br> Ein Jahrhundert Flugzeuge <br> VDI-Verlag GmbH, Düsseldorf 1990, ISBN 3-18-400816-9 |
| [Br80] | Bruß, H . <br> Elektroflug <br> Franzis-Verlag München 1980 |
| [ $\mathrm{Br} /$ Frie98] | Brenner, Th. und U. Friedemann <br> Programm zur Berechnung der Geschwindigkeitspolare unter Einbeziehung windkanal-vermessener Profile <br> Vortrag auf DGLR- Nachwuchstagung 1998 Bremen |
| [Brü21] | Brückner, N., persönliche Mitteilung 1921 |
| [Bü10] | Büchi, R. <br> Faszination Quadrocopter <br> Verlag für Technik und Handwerk, Baden-Baden, 2010 |

[Bü11] Büchi, R.
Brushless-Motoren und -Regler
Verlag für Technik und Handwerk, Baden-Baden, 2011
[DJ20] DJI Mavic Mini im Test (drohnen.de) gepr. 2020
[EK07] Elle, T. und M. Klein
Luftbilder aus dem Modellflugzeug - Untersuchung zu Einsatzmöglichkeiten in der Landwirtschaft
Landeswettbewerb Sachsen „Jugend forscht" 2007, eingereicht von Jugendlichen des Modellflugclub Rossendorf e.V.;
Vortrag auf der DGLR-Nachwuchstagung in Berlin, 2007
[F3K-For] Drehwurftechnik SAL leichter gleich höher? steiler gleich höher?
https://www.rclineforum.de/forum/board19-fl\�\�chenflugmodelle/board27-segler/board86-hlg-sal-dlg/75448-drehwurftechnik-sal-leichter-gleich-h\�\�her-steiler-gleichh\�\�her/ gepr. 2021

| [Gerh] | Gerhard, W. <br> http://www.wolfgang-gerhards.de/zip/schwerpunkt.pdf gepr. 2021 |
| :---: | :---: |
| [Gie15] | Giese, G.: Das kleine LiPo-Abc, http://www.elektromodellflug.de/ gpr. 2019 |
| [Hack19] | https://www.hacker-motor-shop.com/ > A10-13L |
| [Hack21] |  |
| [HebH10/20] | Hebbeker, P. <br> Test: EasyStar 3 von Multiplex <br> FMT, H.10/2020, S. 100 <br> https://www.multiplex-rc.de/ |
| [Hep18] | Hepperle, M. <br>  |
| [HK85] | Haftmann, B; Kiekebusch, B. <br> Details zur Optimierung des Reiseflugwiderstandes des Airbus A320 DGLR-Jahrbuch 1985, S. 89-1 ... 89-36 |
| [HL10] | Hapag-Lloyd-Flug 3378 - Wikipedia |
| [Ho81] | Hofmann, R. <br> Überlegungen zum Kräftespiel beim Hochstart Flug- und Modelltechnik (FMT), Baden-Baden, H.4/1981, S.342-344 |
| [Jant2b] | http://www.piotrp.de/SZYBOWCE/dszd42.htm |
| [Jant78] | Firmenschrift <br> Flughandbuch des Segelflugzeuges SZD-42-2 JANTAR 2B <br> Ausgabe II vom 14.11.1978 <br> Szybownictwa „PLZ Bielsko" |
| [Jä40] | R. Jäschke <br> Flugzeugberechnung <br> R.OIdenburg-Verlag München und Berlin 1949 |
| [Ju65] | Just, W. <br> Flugmechanik <br> Verlag Flugtechnik Stuttgart 1965 |


| [JW04] | Jach, F. und S. Wolrab <br> Entwicklung eines Leistungssegelflugmodells <br> Vortrag auf DGLR-Nachwuchstagung, Dresden 2004 <br> eingereicht von Jugendlichen des Modellflugclub Rossendorf e.V. |
| :---: | :---: |
| [Kra19] | Izabela Krawczyk <br> https://www.aeroclub-nrw.de/tag/zdf/ |
| [KN77] | Kopenhagen, W; Neustädt, R <br> Das große Flugzeugtypenbuch transpress, VEB Verlag für Verkehrswesen, Berlin 1977 |
| [La91] | Langenhagen, H . <br> RC-Segelflugmodelle <br> Entwurfsmethodik, Technologie, Flugpraxis <br> eingereicht bei Transpress VEB Verlag für Verkehrswesen, Berlin 1985 <br> mit Status Satzreife gekündigt 1991 |
| [La94] | Langenhagen, H. und R. Zimmermann Messungen am fliegenden Fernlenkflugmodell - ein Stück experimentelle Flugphysik FMT-Kolleg Nr. 17, 1994, Verlag für Technik und Handwerk GmbH, Baden-Baden |
| [La16] | Langenhagen, H . <br> Nachwuchsgewinnung im Schülerbereich - Rossendorfer Versuche Vortrag zur Übungsleiterschulung Modellflug, Haus der Luftsportjugend Laucha, 03.2016 http://mfc- <br> rossendorf.de/fileadmin/mfcr upload/mfc/Angebote/Schulen/Erfahrungen/nachwuchsSch\%C3 <br> \%BCler c.pdf |
|  | Langenhagen, H . <br> Recruiting pupils for Aeromodelling <br> Lecture at the Annual Meeting of FAI Aeromodelling Commission (CIAM), 04. 2016, Lausanne http://mfc- <br> dorf.de/fileadmin/mfcr upload/mfc/Angebote/Schulen/Erfahrungen/recruitment EN.pdf |
|  | Langenhagen, H . <br> Report: Modellflug in der Schule <br> www.vth.de/fmt/fmt-blog/lorem-ipsum-dolor-sit-amet-consetetur-sadipscing-elitr <br> FMT-online 18.07.2016 |
| [La18] | Langenhagen, H . <br> Lift off from the take-off trolley - uplift force without side effect FAI-CIAM-Meeting/SC Education, April 2018, Lausanne https://www.mfc-rossendorf.de >CIAM Flyer zu unserem Schulprojekt >Startwagen_EN |
| [La19] | Langenhagen, H . <br> Aeromodelling and Aeronautic - A review through History and Physics of Aeronautic -FAI-CIAM-Meeting/SC Education, April 2019 https://www.mfc-rossendorf.de > News >Unser Wort bei der CIAM in Lausanne >Anhang_EN |
| [Lab 08] | MIT OpenCourseWare Lab 8 Notes - Basic Aircraft Design Rules 6 Apr 06 -https://ocw.mit.edu/courses/16-01-unified-engineering-i-iii-iii-iv-fall-2005-springff8a4985ee6ea819afb90c40834e3 spl8.pdf |
| [LeSol] | https://lemo-solar.de/shop/modellbautechnik.php (alter shop, wird nicht mehr gepflegt) https://shop.lemo-solar.de/ (neu) |
| [LH] | Lernhelfer/Schülerlexikon > Physik > Gleichstrommotor https://www.lernhelfer.de/schuelerlexikon/physik/artikel/gleichstrommotor\# |


| [MB10] | http://www.urlaub-und-hobby.de/metallbaukasten/so04dt.html > sonstige Projekte > Kenngrößen von Gleichstrommotoren |
| :---: | :---: |
| [Mu78] | Musil, M. |
|  | Aerodynamika modernich leteckých modelu Prag, 1978 |
| [Mü14] | Müller, M. www.eCalc.ch |
|  | Elektrisch, aber richtig; Artikelserie in Aufwind 2014/H4 bis 2016/H2 |
| [MPX21] | Roxxy-Antriebsfibel https://www multiplex-rc de/Downloads/Multiplex/Kataloge/Roxxy-Motoren-Fibel-web pat |
|  | März 2021 |
| [ $\mathrm{NaWi14]}$ | Nacht der Wissenschaften 2014; Poster Leibniz-Institut für Polymerforschung Dresden |
| [Neu67] | Neunaß, E. |
|  | Praktische Strömungslehre |
|  | VEB Verlag Technik Berlin, 1967 |
| [Pi16] | https://shop.pichler.de/Startseite > XQ-Card gepr. 2016 |
| [Reis21] | https://www.reisenauer.de/aid-2195-Leopard-LC250-30-6pol-5550KV-br-mit-Micro- |
| Edition-5-1NL-T.html gepr.04. 2021 |  |
| [R\&G20] |  |
| [Sc13] | Schmidt, F. |
|  | Kräfteeinwirkungen am Flugmodell |
|  | - welche Kraft muss eine Rudermaschine wirklich aufbringen? - |
|  | Landeswettbewerb Sachsen „Jugend forscht" 20013, eingereicht von Jugendlichen des Modellflugclub Rossendorf e.V |
| [Sche02] | Schenk, H |
|  | Abschätzung des Antriebs von Elektroflugmodellen https://www.rc-network.de/threads/absch\%C3\%A4tzung-des-antriebs-von- |
| elektroflugmodellen.11779892/ |  |
| [Sche03] | Schenk, H. |
|  | Der Standschub von Propellern und Rotoren |
|  | http://www.rc-network.de/magazin/artikel 02/art 02-0037/Standschub.pdf |
|  | gepr: Jan. 2011 |
| [Sche07] | Schenk, H. |
|  | PropCalc 3.0 für Windows; - Einführung, Fortschrittsgrad\&Co, Prop und Motor http://www.drivecalc.de/PropCalc/index.html gepr: Jan. 2011 |
| [Schu80] | Schulze, Horst |
|  | Luftschrauben für Modellantriebe |
|  | Transpress VEB Verlag für Verkehrswesen, Berlin 1980 |
| [Se89] | Selig, M., Donovan, F. and Fraser, D. B. |
|  | Airfoils at low speeds |
|  | Soartech 8, H.A.Stokely, publisher |
|  | 1504 North Horseshoe Circle, Vorginia 23451, USA |
|  | http://airfoiltools.com/airfoil/details?airfoil=s7012-il gepr. 2020 |
| [Seid62a] | Seidler, Fritz |
|  | Die stationären Flugzustände |
|  | Franksche Verlagsbuchhandlung, Stuttgart 1962 |

[Seid62b] Seidler, Fritz
Die Stabilität in der Flugmechanik Franksche Verlagsbuchhandlung, Stuttgart 1962
[SI] Mit den Flügeln der Sonne -- Solar Impulse 2: ohne Treibstoff um die Welt https://www.scinexx.de/service/dossier_print_all.php?dossierID=91238 gepr. 2021
[SLZ77] Schulze,H., Löffler, A., Zenker, W. Modellflug in Theorie und Praxis transpress VEB Verlag für Verkehrswesen, Berlin 1977
[Sz42] Schmitz, F.W.
Aerodynamik des Flugmodells
Luftfahrt-Verlag Walter Zuerl; Steinebach -Wörthsee, 6. ergänzte Auflage, 1975


[^0]:    ${ }^{1}$ In commercial aircraft operations, unpowered flight does not normally occur. Descent flights are always with the

[^1]:    ${ }^{2}$ Since the drag force is only about $1 / 10$ of the lift force, we neglect its effect here. Also, for the sake of clarity, the pressure force acting on the underside is omitted in the sketch.
    H. Langenhagen

[^2]:    ${ }^{3}$ the use of approximation functions for the computational estimation of technical interrelationships is very expedient. Since it is not a matter of high accurcy, their solutions are referred to as a qualitative progression.

[^3]:    ${ }^{4}$ Wild geese, for example, utilise the effect of the upward flow behind the wingtips when flying in wedge-shaped formations. This saves them energy.
    On the other hand, the wake vortices caused by large commercial aircraft can be dangerous for smaller sports aircraft if they only follow at a distance of 1 to 2 kilometres.

[^4]:    ${ }^{5}$ This is the case for $R \_7=0,63$ Ohm / kv_7 $=1540 / V m i n ; R \_10=0,66$ Ohm $/ \mathrm{kv} \_10=1335 / \mathrm{Vmin}$
    ${ }^{6}$ Instead of $7,4 \mathrm{~V}(2 \mathrm{~S})$ und $11,1 \mathrm{~V}(3 \mathrm{~S})$ the values 7 V and $10,5 \overline{\mathrm{~V}}$ are common as test voltages

[^5]:    ${ }^{7}$ To be precise, the wing mass $m_{w}$ does not play a role in load multiplier considerations, since the wing is self-supporting, so to speak. However, the total model mass is often used

[^6]:    ${ }^{8}$ The time curves are solution curves of a differential equation that describes the starting process, see also section 8.8

